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14. ABSTRACT

Global awareness (GA) entails the acquisition of data from local to global levels, appropriate fusing of the data, and presentation of that data as useful information. This data will then be fused to fully describe situations of interest such as large transportation systems and complex communication systems. This project specifically aims at developing the mathematical basis, architecture and software techniques implementing particular new technologies to support Global Awareness and comprises six main tasks. Task 3 was:
3. Advanced Technique for Search Problems of Moving Objects;

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Project # 1993P
Mathematical Basis of Knowledge Discovery and Autonomous Intelligent Architectures
Task # 3
New Class of Search Problems for Moving Objects

Final Report

Principal Investigator
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St. Petersburg

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Content

Preface

This document is the final Report on task #3 « New Class of Search Problems for Moving Objects» of the Project # 1993P «Mathematical Basis of Knowledge Discovery and Autonomous Intelligent Architectures» that is being carried out according to the agreement between European Office of Aerospace Research and Development (EOARD), The International Science and Technology Centre (ISTC) and St. Petersburg Institute for Informatics and Automation of the Russian Academy of Sciences (SPIIRAS). The Report contains results of two years research done according to the Work Plan.

According to the Work Plan three general steps have been completed as follows:

1. An axiomatic basis for theory of search for moving objects was developed. In this regard a broad spectrum of theoretical manuscripts, papers and monographs was examined. A classification of search tasks, determination of language, main abstractions of the theory and system of axioms were developed.

2. A common theorem of additivity and theorem of multiplicity for search potentials of different observation systems were developed.

3. A computer interpretation of search tasks that includes a library of functions and visual components and some search tasks was developed.

The Work Plan is fulfilled completely.

Task #3 Principal Investigator
Professor Vasily V. Popovich,
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Glossary

Observation system	A technical system for the Target detection.
Observation zone	A zone around the Observer (Target) where the probability of the Target detection is over zero.
System of observer's physical fields	A set of factors that affect the Region.
Set of variants of observer moving	A set of motion variants to cross the Region.
Operation	A system of activities to achieve the goal of search – detection of the Target.
Observer	An object that intends to find the other object – Target.
Target	An object that is the goal of search.
Region	A place (sea, land, forest or set of conditions) where the Target and Observer are located.

PATL	Probability area of target localization.
FSO	Field system of observer.
EWST	Effective width of asearch track.
ZPFR	Zone of physical field registration.
Set of variants of observer's system activities	A set of Region monitoring methods for the Target detection.

Introduction

The theory of search (TS) was developed during a short time period when after the Second World War some reports of the USA scientists were published. They considered results of research completed during the Second World War, and currently they are well known as the reports by B.O.Koopman [21-23]. Rather soon many different theoretical approaches were developed based on them and followed by the methods of operations research. Most of them are given in Table #1. Each method as shown has some advantages and some limitations, and each method as a rule is used for solving some application tasks.

Table # 1. **METHODS OF OPERATIONS RESEARCH**

Modeling (Analysis)

Analytical methods			Statistical methods	
Markovian discrete chains	Queueing theory	Theory of search	Monte Carlo method	Sequential analysis method
RANGE OF APPLICATION				
System management. The system can change its states during some time.	The wide class of stochastic processes with denumerable number of states and continuous time of transition modeling.	Mathematical modeling of search for different targets.	It is only limited by a computer power. As a rule it is used when all probability properties of the process are well known. It is used when: - analytical methods are not available; - limitations of the analytical methods distort a research object; - some limitations of the analytical methods need to be checked.	It is used to evaluate an object depending upon the quality specifications. Or when two objects are being compared, and the best one should be chosen.
LIMITATIONS				
- it is necessary to know (to determine) probabilities of system's transitions; - it is difficult to create models for equations transitions' probabilities; - Markovian discrete chains are chains not returning to the previous state; - big number of	-it is used only to queueing processes; - main properties such as: stream of requests, channels of operating, queue discipline should exist; - probability properties of requests' stream and time of	- it is used for search of objects; - aftereffect is not taken into account; -target is passive (it does not avoid detections).	- it takes too much time to realize large models on PC; - it is difficult to optimize complex processes on PC; - developing such models on PC takes much time; - interpretation of such models is a very complex problem, it is not solved yet.	- it is necessary to estimate a value of mistake of the first and the second type; - it is necessary to create a special experiment with a limited number of tests; - it is hard to determine demands to a new model, method, etc..

states complicates equations; - time of transition is discrete; - denumerable number of the system states.	operating have to be stationary, ordinary, without aftereffect.			

Optimization (Synthesis)

Linear programming	Nonlinear programming	Dynamic programming	Game theory	Statistical decision theory	Network planning method
RANGE OF APPLICATION					
Efforts, facilities, and their distribution along tasks or objects. Basis for a method of the efforts and facilities use.	The range of application is the same as in linear programming. It is used when goal function and limitations are given as nonlinear.	Optimization on the efforts and facilities, and their distribution along tasks and objects with a choice of a method of efforts and facilities use.	- possible variants of player's actions evaluation; -rational variants of player's efforts and facilities development ; - manuals and help system on efforts and facilities used for players modeling; -decision making under the conflict situations.	Cases when efficiency of efforts and facilities used depend on many undeclared conditions.	Plan, organize and control efforts and facilities.
LIMITATIONS					
-goal function and limitations have to be linear; -it is difficult to declare a goal function, limitations and suppositions.	In comparison with linear programming this method is not universal.	-a process of the plan development have to be interpreted as Markovian process with system's transitions from the first state to the end state. The end state provides for the maximum of the goal function; -attribute of efficiency has to be a sum or a composition of attributes of all steps.	The following must be known: -sets of players strategies; -payoffs (losses) for all pairs of strategies; -player's reason for a choice of the optimal strategy. Modeling results can be used in the next cases: -multiply repeated lots of the same game; -multiple variants during one game.	The following must be known: -hypothesis about possible conditions at the moment when a variant of the plain is being realized; -probabilities of the above hypothesis. Each act and set of conditions must have an attribute of efficiency.	It is used for processes planning, using and controlling, when the processes include over 100 actions.

As we can see in Table #1, disregarding the set of operations methods, TS is used for solving some particular tasks. Numerous well known attempts to receive new results in TS as well as many publications in this field did not contribute to TS. Only few of them really developed some applications based on classical TS methods. It happened because search tasks first of all have physical rather than mathematical nature, so modern algebra, probability theory and other formal

theories is only the instrument but not the independent subject of research in the theory of search. The fact is that some well-known monographs and articles are rather simple from the mathematical point of view or do not belong to TS.

The goal of our project is not a selection, classification or enumeration of all published manuals, monographs, articles, tasks and so on about TS, it is sooner developing a system of rules that we call “Theory of search for moving objects”, developing the New Class of Search Problems for Moving Objects within the frames of this system. According to this thesis details of the Work Plan were planed.

1. Development of the axiomatic basis for TSMO.

Judging by Kim [45] and disregarding many publications that appeared after articles by Koopman, theory of search have not been created yet. In this project only a case of search for moving objects has been studied. The reasons why this class of search was selected are as follows:
1) it is necessary to determine limitations within which one can guarantee theoretical integrity, completeness and consistency for well-known and new TS problems;
2) it is necessary to interpret quantitative data for real operations of search;
3) computer interpretation of theoretical results is necessary.

With due regard to the above reasons we have planned our research in the following order:
1.Axiomatic approach for TS (creating TSMO).
2.Numerical interpretation of theoretical results.
3.Computer (object-oriented) interpretation of TSMO.

Let us give an analysis of publications on TS and highlight the important tasks of TS.

1.1. Analysis and classification of search problems.

The basis of the theory of search was developed by B.O.Koopman, though his results were published some time later [21-23]. Papers written by Koopman were important bases for the other authors. Many researchers working in the theory of search developed the main Koopman problems and created certain new approaches, such as Game Theory and others.
Some main problems were divided into three groups and their consideration was published by Koopman in three reports as follows:

1. Kinematic bases;
2. Target detection;
3. The optimum distribution of searching efforts.

1.The following problems were described in his first report:

- 1.1.The analytical description of equations of target and observer moving (under constant course and velocity);
- 1.2.The analytical description of equations of connecting region and probability estimated value of connecting an observer and a target;
- 1.3.The analytical equation describing the randomly distributed targets;
- 1.4.The analytical equation describing the evenly distributed targets;
- 1.5.The probability estimation of the randomly distributed targets;

2.The following problems were described in his second report:

2.1.The analytical description of instantaneous probability for target location;

2.2.The analytical description of location probability (depending on targets and observers tracks);

2.3.The analytical description of horizontal distance distribution;

2.4.The analytical description for a common case of a random search;

2.5.The analytical description for a particular case of parallel sweeps;

2.6.The analytical description of the forestalled detection of a target by an observer.

3.The problem of optimum distribution of search efforts was considered in his third report.

The problems later considered by Koopman and other authors in the articles and books could be conditionally divided into two groups:

1.Formal-algebraic research.

2.Applied research.

The formal-algebraic research forms an approach to formal extension of search problems (e.g., [48]). Unfortunately well-known studies in this field do not include very important results of the theory of search, because search tasks have physical rather than mathematical nature, so modern algebra, probability theory and other formal theories is only the instrument but not the independent subject of research in the theory of search.

The applied research was more successful in the sense of TS development

V.P. Lapshin [46] formulated the above mentioned 1.4 problem as a problem of probability density distribution of the targets being detected by the observer bearings. Also he gave a complete analytical proof of this problem. As a consequence, a problem of probability for target location detecting in an interval of observer bearings, and the problem of probability for target location defining in an observer bearing were also described. The Koopman’s problem of optimum distribution of search effort was used to develop an algorithm [46] for a computer model run on PC.

V.A. Abchuk et al [1] showed some practical applications of search problems. In [1] some new results in operations research that were obtained at the end of 70-s, and the main problem of Game Theory of search by Isaacs [3] are given. Problems of the Game Theory were further developed in the works of Zenkevich and Petrosian. [1] also shows, that an interpretation of the observer location zone as a circumference is not correct for the most of real conditions. Unfortunately, a solution of 2.4 problems was not correct for all cases. It is correct only for a case where velocity of the target is close or equal to zero.

Charnes A. and Cooper W. [9] solved the problem 3 as a problem of the convex programming.

McQueen J., Miller R.G. [29] formed problems for decision making for search tasks. First, is it reasonable to start the search under real conditions, second, is it reasonable to continue the search after a definite period of time or to stop it? The common functional equation was derived.

De Guenin J. [12] summarized results of the problem 3. He assumed that the detection function can be an arbitrarily fixed function of density. In some other sources [11,13, 28, 36, 38, 39] special cases of the problem 3 are discussed.

Dubrovin and Sirotin [41] solved a problem of the average time of target existence in the rectangular search region, when at the initial time instant of search the target coordinates are subjected to the uniform law of probability distribution. This problem is a more detailed version of the result received by McQueen J., Miller R.G. [29].

Corwin T. [10] considered the problem 2.4 (definition of the target detection probability) for a case when targets' coordinates are described as the Wiener process in a phase space R_1 and R_2. In [30] it is shown that the target detection probability does not depend on the initial target's coordinates (confirmation of the Koopman's inference).

V.A. Gorbunov [40] studied in detail problems 2.1 and 2.3. He suggested a semi-empirical algorithm for the definition of the effective track width for the target search.

D.P Kim [45] suggested a general statement for the problem of search.

Let the search process be described as follows:

$$\dot{z} = f(z, u_{\Pi}, u_{\mathrm{N}}, \xi, t), \qquad t \in \lfloor t_0, t_f \rfloor, \text{ where}$$

$z(t_0)=z^0; \quad u_{\Pi} \in U_{\Pi}; \quad u_{\mathrm{N}} \in U_{\mathrm{N}};$

z–state vector, it includes phase coordinates of the target and observer, and their kinematic properties;

u_{Π}, u_{N} –controls of the observer and target;

ξ –vector of the probability process;

U_{Π}, U_{N} –classes of allowable controls u_{Π}, u_{N}.

The search is shown inside the time interval $t \in \lfloor t_0, t_f \rfloor$.

It is needed to define a search algorithm (control algorithm) for the target and the observer under the conditions as follows :

$$J_1 = J_1(z, u_{\Pi}, u_{\mathrm{N}}, t) \to \max_{u_{\Pi} \in U_{\Pi}} \cdot$$

$$J_2 = J_2(z, u_{\Pi}, u_{\mathrm{N}}, t) \to \max_{u_N \in U_N} \cdot,$$

where: J_1, J_2 – given functions of functions.

In [45] Kim proved that such a statement of the problem indeed is a common problem of search, and that it also has no practical application. For a more detailed description Kim suggested to study three different theories of search: discrete search, continuous search, and game search. Such a detailed elaboration is very important since the operations research considers the search as a retrieval of any kind of information. From this point of view many of the conventional approaches, such as mathematical statistics, mathematical programming, calculus of variations and other can be identified as the problems of search.

The analysis of the second group of search problems shows that some problems of the advanced theory of search are not solved yet. The main effort of researchers was focused upon solving the problem 3, that is similar to mathematical programming and calculus of variations. The problems of the target detection have not been solved. These problems can be described as follows:

1) Taking into account an interval of the observation zone. This case was described in [45] accounting for simple conditions (a velocity and a course of target and observer are constant). For a general case that problem was considered in [33]. In [1] a solution of this problem was given only for a simple case.

2) The case of the complex observation system integrated within one carrier. A variant of this problem was shown in [22] in a form of a theorem of additivity but only for similar observation systems. The case for multi-type observation systems was shown in [34,35] and more details were given in [33].

3) Estimation of effectiveness of a large target or a track of target was shown in [22]. In more detail it was shown in [33] and [34]. In [33] an approach to building an axiomatic theory of search was discussed. In the proposed case the group of problems of "continuous search" is constricted to the group of problems of "moving object search". First, a conceptual problem of search is formed. It is similar to the problem of "The beauty and the beast" suggested by R. Isaacs. After that theory (not a problem) of search for moving objects is being formed.

Let the theory be some nonempty set as follows $\mathbf{T}\langle \mathbf{L}, \mathbf{A}, \mathbf{H} \rangle$, where

L – language of the theory;
A - axioms of the theory;
H - theorems of the theory.

At the next step, a set of search problems, that are correct in this case, is described. A change of axioms is possible. It is a good way to extend a basis of the theory. The same version was selected for introduction of new classes of search problems for different kinds of observation systems integrated within one carrier.

1.2. Forming axiomatic basis of the research approach.

Processes of search for moving objects, when an observer or/and a target change their location, form the *subject of inquiry*.

The tasks of creating and versatile studying the search models, both mathematical and computer, form the research *subject-matter*.

As the *abstract task statement* the "beauty and the beast " task as formulated by R. Isaacs was chosen. There is a room (environment or medium) big enough as compared with a size of the beast (an observer) and the beauty (a target). The beast and the beauty can move unrestrictedly inside the room, not knowing about the location of each other. It is required to develop an analytical model of search to evaluate its efficiency. The following criteria describe the search efficiency:

- the search time needed to find the beauty;
- the probability of the beauty detection.

The following attributes of search efficiency are used: $P(t), \bar{t}$, where:
$P(t)$ - probability of target search, during some time (t);
$\bar{t}$ - average of distribution for search time.

For the common case three main objects could be selected, they form a problem of search, such as: Observer, Target and Region. Observer is an object that intends to find another object – Target. Target is an object that is a goal of search. Region is a place (sea, land, forest or set of conditions) where Target and Observer are, see Fig.1. For our case let Region be a square. According to the object-oriented approach (OOA) Observer, Target and Region will be interpreted as classes. Let us add one more class "Operation" to three main classes. Operation is a system of activities directed to achieve the goal of the search. In comparison with the other classes (Observer, Target and Region) the class Operation can be interpreted as a meta-class, because it includes the others' classes, see Fig.2.

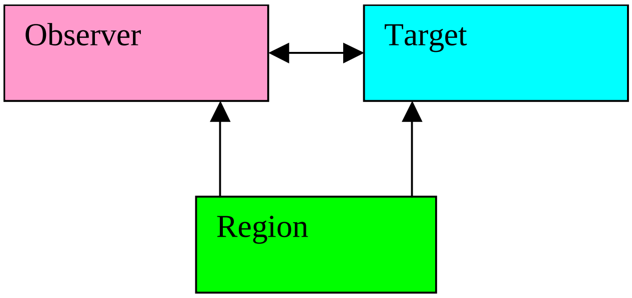


Fig.1. The main objects of search.

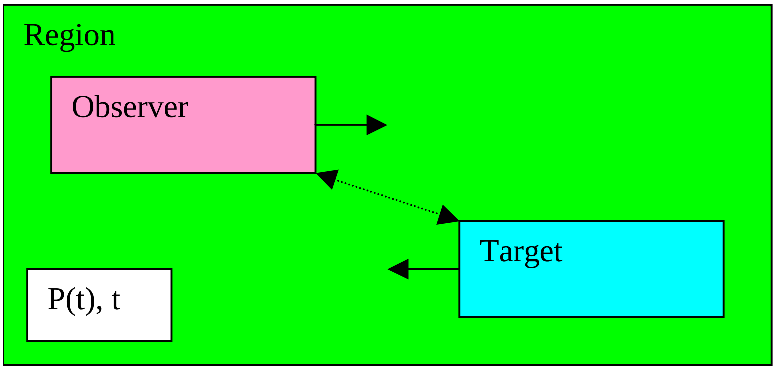


Fig.2. The Operation meta-class.

The meta-class Operation can be interpreted as a vector:

Operation=(Observer, Target, Region).

Following the OOA, each class should be represented as a set of objects. For that we should find a classification base. For example, let Observer be divided into some parts, as follows (see Fig.3):

1) observer system (s) for detection of physical fields of the Target;
2) set of variants (possible) of observer system activities;
3) set of variants (possible) of observer moving;
4) system of observer's physical fields.

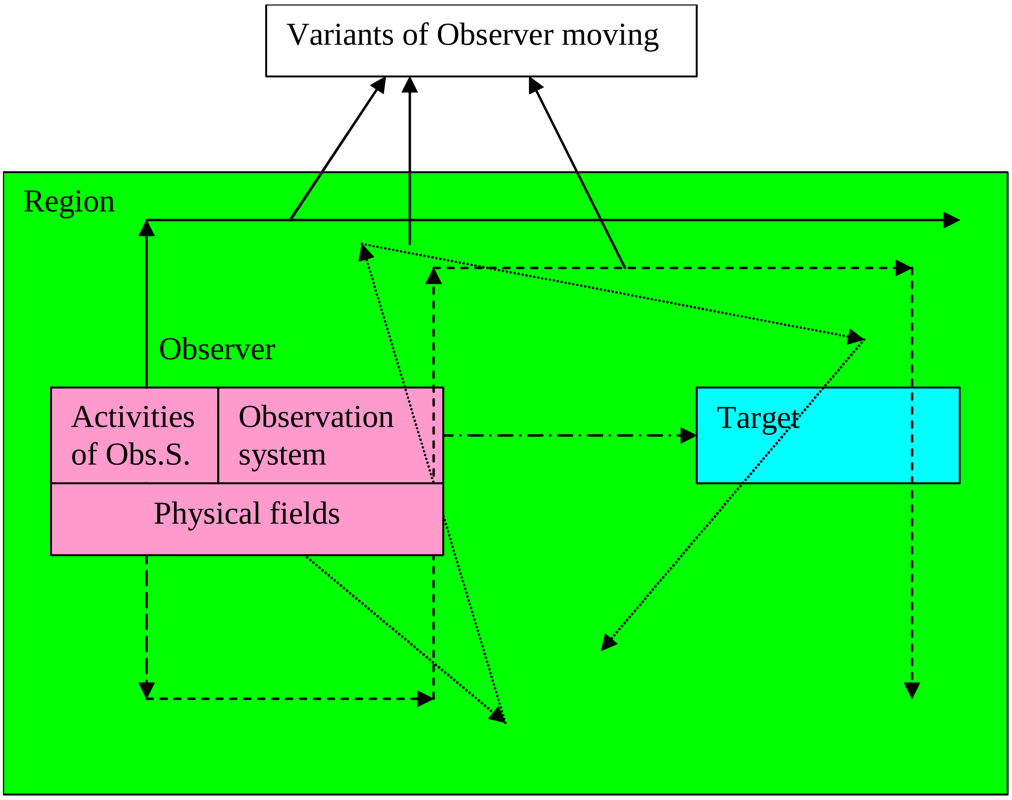


Fig.3. Observer class in a search operation.

In many cases, we can consider the Observer as an observation system(s). The main goal of the observation system's activities is the detection of the Target. So, in our case Observer and observation system are synonyms. One can notice that observation system is a necessary and sufficient condition for the Observer's existence. Depending on different causes the other Observer's attributes can be absent.

Let us introduce some definitions.

The set of variants of observer system activities is a set of the Region monitoring methods for the Target detection.

The set of variants of observer's moving is a set of move's variants for crossing the Region.

The system of observer's physical fields is a set of factors, that affect the Region.

The Target class can be described like the Observer.
1) observer system (s) for detection physical fields of the Observer;
2) set of variants (possible) of observer system activities;
3) set of variants (possible) of Target moving;
4) system of Target's physical fields.

The classes defined above describe our research field. This level can formalize the research field, however, it is only a theoretical level. Next level representation of the research field can be determined as a description of real search operations, and the simulation and imitation methods will be better for this application. The simulation and imitation methods are useful when results of search activities in the real fields such as the search within geographic localities exposed to extreme conditions, like forest fires, floods, and other caused only by force majeure circumstances are well known. It is strongly recommended that the theoretical level must go prior to statistical and other methods of research.

So, at this step, the conceptual model of search has been described. At the next step the conceptual model of search will be formalized. The results of formal algebra (axiomatic method) will be used for that.

Forming the theory alphabet.

For the formal algebraic representation of the theory let us use the theory representations developed in the mathematical logic and in the theory of models.

A certain not empty set $\mathbf{T}\langle \mathbf{L}, \mathbf{A}, \mathbf{H} \rangle$, where:

L – language of the theory (in our case we operate with typical algebraic symbols of the mathematical analysis, theory of probability, algebra-logic, their syntax and semantic);

A – axioms of the theory (sentences (formulas) considered true and not being proved within the given theory);

H – theorems of the theory (true sentences), being proved based on the language syntax, its semantic and axioms, will be understood as the theory.

Besides the formal language symbols the conceptual sets of the language of the theory will be considered, such as: **Observer, Target** , **Region, Operation**.

Let us examine those sets.

Observer = (K_n, V_n, D_{ef}), where:

K_n – a course of the observer (in degrees), can be determined as a constant, as an interval, as a stochastic variable;

V_n – velocity of the observer (in knots), can be determined similarly to the course;

D_{ef} - effective width of a search track – it is a parameter, characterizing a search potential of the observer. It is a function of function as follows:

$D_{ef} = f(V_n, T\arg et, \mathrm{Re}\, gion)$.

So, we can see interconnection between all operation objects.

Target = (K_c, V_c, D_{efc}), where:

K_c - course of the target (in degrees), can be determined as a constant, as an interval, as a stochastic variable;

V_c - velocity of the target (in knots), can be determined similarly to the course;

D_{efc} - effective width of a search track – it is a parameter, characterizing a search potential of the target. It is a function of function as follows:

$D_{efc} = f(V_c, Observer, \mathrm{Re}\, gion)$.

Region = (a, b, S, K), where:

a – a search region width (in miles);

b – a search region length (in miles);

S – a search region square (S=a*b);

K – set of region auxiliary properties such as acoustic fields, temperature, wind and so on.

Operation = (G, F), where:

G – a set of operation hypothesis (detecting a target class, detecting a physical target field, detecting the target presence in the region of search, etc.);

F – set of properties, that could not be described as a mathematical model and can influence upon the results of search.

The analysis of search problems shows that the axioms set A is infinity. Practically every new task of search introduces a certain statements, which cannot be proved within the previous statements and uses the results and methods of other theories without any proof. Currently we cannot consider the theory of search to be a unified theory.

1.3 Forming the axioms of theory of search.

Based on results, received at the previous steps we will form the set **A** of axioms (see above), demonstrating the applicability boundaries of the approach being developed. This will allow to expose certain problems that were not posed before, and not to consider certain problems beyond the frames of the proposed axioms under the TSMO (e.g., game theory). It is so because it is impossible to make a universal system of axioms for any task of search. Some times when the study of a new task begins, the old axiom systems must be amended or changed.

The first (classical) axiom system.

1.Let randomized phase coordinates be evenly distributed:

1.1.The probability that $\mathbf{K_c} \in (\varphi_1, \varphi_2)$ is $\mathbf{P_k} = \frac{\varphi_2 - \varphi_1}{2\pi}, \varphi_1 < \varphi_2$.

1.2. If the Target is within some region **B**, then the probability of its being within the very small region $\mathbf{A} \subseteq \mathbf{B}$ if **A<<B** is $P_p = \frac{S(A)}{S(B)}$, where S(A),S(B) –**A** and **B** squares correspondingly.

1.3. Events that give rise to probabilities $\mathbf{P_к}$ and $\mathbf{P_p}$ are mutually independent.

2. Let $\mathbf{V_c}$ be nonprobabilistic variable and $\mathbf{K_c}$ a random variable that is evenly distributed over an interval $(\mathbf{0,2\pi})$.

3. Isolated values of $\mathbf{K_n}$ are random and independent.

4. Observation zone $\mathbf{D_{ef}}$ is a variable that is an integral attribute of Observer's observation system. If the Target is in the observation zone (to the left or to the right of Observer) the probability of Target's detection will be = 1.

5. The process of the Target's detection is the Poisson process, for which:

- stationary process (the probability of "n" detections over the $(\tau, \tau + t)$ interval of time independent of τ);
- process independent of increases («without aftereffect») (the probability of "n" detections over the $(\tau, \tau + t)$ interval of time independent of the number of detections before this time interval);
- ordinary process (detection probability more then one target over a $(\tau, \tau + \Delta t)$ infinitesimal interval is a superior infinitesimal variable than Δt).

6. The Target's reaction is set aside directly, and it can be taken into account indirectly. The tasks where the Target's reaction is set will be considered as tasks of the game theory.

The axioms selected above are not included into TSMO. They are correct sentences of the other theory(s), the formal algebra and the theory of probability for our case. In this case we can assert that the axioms' system is a containing system of axioms (a formal theory is a theory where axioms are declared in the frames of the same theory and their correctness cannot be proved by any other theory) [44]).

The basic theorems of TS can be proved on the basis of the axioms' system. Let us give definitions of these theorems. A full proof of the new theorems will be given somewhat later in this report.

The main theorem of TS. The effectiveness of the Target search by Observer in the Region of search can be evaluated as: $P(t) = 1 - e^{-F(t)}$, where
F(t) – the search potential of Observer.
At first this was proved in [12].

Lemma 1. The effectiveness of the Target search by Observer can be evaluated by a determined variable – an average of distribution of search time in the Region: $\mathbf{MO_t = \frac{1}{\gamma}}$, where
γ - intensity of Target search. This proof is given in some books, for example, in [14].

The theorem of additivity. The search potential of N independent Observers has a property of additivity. If in a Region of search the N (N>=1) independent Observers are looking for the Target the probability of Target detection is: $P(t) = 1 - e^{-\sum_{i=1}^{N} F_i(t)}$. The theorem had been proved in [22].
The search potential is: $F(t) = \frac{Ut}{S}$, where
U – the search capacity of Observer (square miles per hour);
t – the search time (hours);
S - a search region square (square miles);

$U = D_{ef} V_\rho$, where $\mathbf{V_\rho}$ - mean relative velocity of the Target.

Lemma 2. The mean relative velocity of the Target can be determined as:
$\mathbf{V_\rho = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{V_n^2 + V_c^2 - 2V_c V_n Cos\theta d\theta}}$, where θ - the angle of the course of the Observer with the course of the Target (this is evenly distributed in $(0,2\pi)$).

In some research, for instance, in [3], the following equation is used for an approximate computation: $\mathbf{V_\rho \approx \frac{V_n^2 + V_c^2}{V_n + V_c} + 0.3\sqrt{V_n V_c}}$.

Our research showed that in some cases if this equation is used one can get a mistake up to 10% of V_ρ.

Calculation of $\mathbf{D_{ef}}$ is a very complex issue because the universal proof for the entire set of Observation Systems does not exist. We only know the proof of the existence theorem, and it is of no use for practical implementations.

The first extension of TS.
There are some problems when an effectiveness of Observation Systems group within one carrier is being calculated. The theorem of additivity for the case can only be used for a particular class of Observation Systems. For different types of Observation Systems this theorem as a rule cannot be used. There are no methods and theorems for solving this problem through the

classical TS. Thus, there is a good reason to extend the TS. There is a good solution to use the theory of tactical maneuver [24]. And it is possible to receive a good solution of effectiveness of different Observation Systems within one carrier. The theorem of additivity is a special case of a common solution. The independence of Observation Systems is a strong condition.

The second extension of TS.
The description of search kinematics showed that the search process possesses some "memory". In other words the fact that the Observer saw the zone in the region and did not detect the Target can be used. In this case the dependence on different Observation Systems during the search process must be kept in mind. If in this case the Target track is detected, some area where the Target is can be determined. This class of tasks is the second extension of TS. This extension will be called the multiplication tasks.

The third extension of TS.
It is the Target search when its coordinates are not evenly distributed. Some time ago such kind of search was called the "search after call". It is the fact that density of coordinates distribution depends on time and as a rule tends to be evenly distributed. Not too many practical cases when the Target's coordinates distribution is independent of time are known.
The proof of the Target coordinates distribution in time theorem and some conclusions are given in [21]. Our computer modeling shows that this theorem cannot be used at PC because the meaning of distribution density tends to zero too quickly. So, if the meaning of time is **t > 3σ,** the meaning of distribution density will be zero, where **σ -** the standard deviation. We have obtained the other algorithm for the distribution density. Let us consider this theorem.

Theorem "search after call". If the target has been detected in some point of the region and:

- Target's coordinates are distributed under a normal law;
- Parameters of target's moving are unknown;
- Target's velocity is over zero;

It is possible to determine density of Target's coordinates distribution at any period of time after its detection as a function:

$$f(x,y)=\frac{1}{2\sigma_y\sigma_x\pi^3}e^{-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}}\int_0^{\pi}e^{\frac{2x\cdot L\cdot Cos\varphi-L^2Cos^2\varphi}{2\sigma_x^2}}d\varphi\cdot\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}e^{\frac{2y\cdot L\cdot Sin\varphi+L^2Sin^2\varphi}{2\sigma_y^2}}d\varphi.$$

Proof.
Let the Target's coordinates be represented as a normal law on the plane and density of distribution is determined as follows:

$$f(x,y)=\frac{1}{2\pi\sigma_x\sigma_y}\cdot e^{-\frac{x^2+y^2}{2\sigma_x^2\sigma_y^2}}$$

It is need to determine a law of Target's coordinates distribution if after detection the Target can move in any direction from **0** to 2π, and a velocity $\mathbf{V_c} > 0$.
Density of distribution along one of a datum lines can be described as follows (Fig.4):

$$f(x)=\frac{1}{\sigma_x\sqrt{2\pi}}\cdot e^{-\frac{x^2}{2\sigma_x^2}}$$

It corresponds to projection of radius of circular distribution on the plane in one direction.

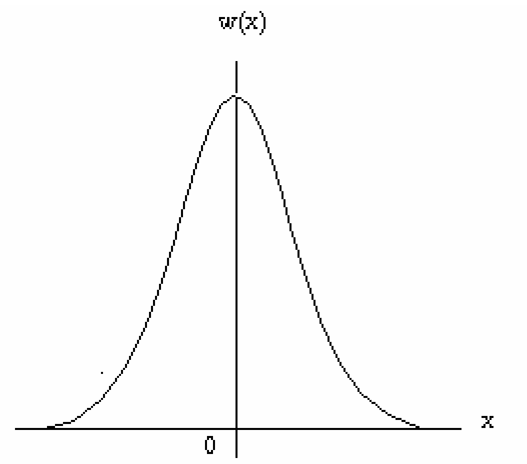


Fig.4. Density of normal law distribution of one Target's coordinate along datum line.

In this case every point **x** is distributed under normal law. Let some point have a meaning **x** (Fig.5).

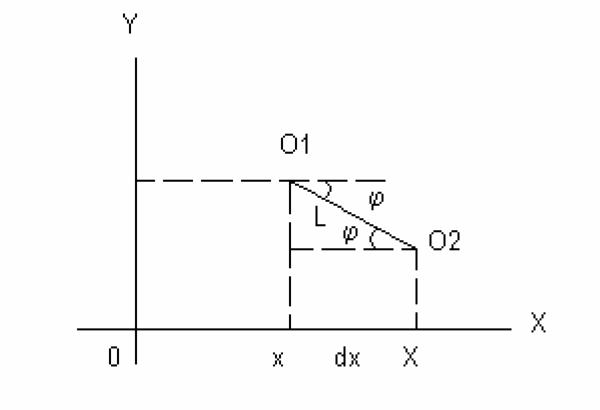


Fig.5.Evolution of Target's coordinate after some time delay.

After some time the coordinate **x** will be changed, because the Target can move in any direction with equiprobability from **0** to 2π. Projection of **x** will be evaluated in an interval of target's courses from **0** to 2π. Its increase can be determined as follows:

$$\boldsymbol{dX = L \cdot Cos\varphi}, \quad \text{where:}$$

L = V_c t_3,

t_3 – a period of time from Target detection to the current situation.

So it is possible to obtain a new meaning of Target's coordinate along the datum line **X** some time after Target's detection as follows:

$$\boldsymbol{X = x + dx = x + L \cdot Cos\varphi} \ .$$

If a law of **x** distribution is known (normal law) and a law of **j** distribution is known too (equiprobably from **0** to 2π), it is possible to determine a law of distribution for new Target's coordinates.

Let **z = L Cos(j);** (Fig.6.). Function **z** is monotone in an interval **-L,L**.

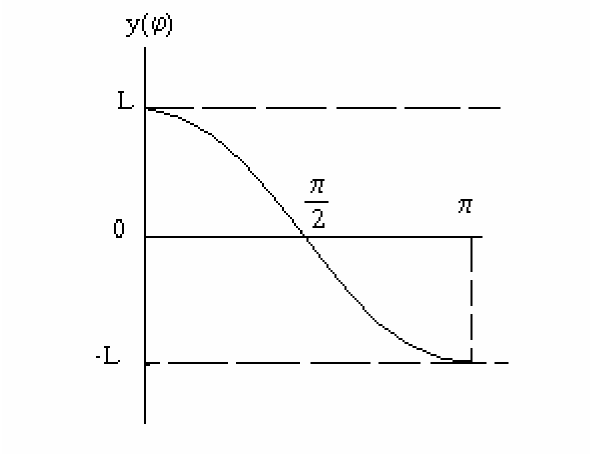


Fig.6. Function **z = L Cos(j).**

For this case it is possible to use a well known in mathematical statistics algorithm for density distribution of random variables system determination as follows:

$$\boldsymbol{f(j)} \Rightarrow \qquad \frac{1}{\pi}$$

$$\boldsymbol{z} = \boldsymbol{f(j)} \Rightarrow \qquad \boldsymbol{z} = \boldsymbol{L} \cdot \boldsymbol{Cos}\varphi$$

$$\boldsymbol{j} = \boldsymbol{y(z)} \Rightarrow \qquad \varphi = \boldsymbol{arccos}\left(\frac{\boldsymbol{z}}{\boldsymbol{L}}\right)$$

$$\left|\psi(\boldsymbol{z})'\right| \Rightarrow \qquad \frac{1}{\sqrt{1-\left(\frac{\boldsymbol{z}}{\boldsymbol{L}}\right)^2}}\frac{1}{\boldsymbol{L}} = \frac{1}{\sqrt{\boldsymbol{L}^2-\boldsymbol{z}^2}}.$$

Then:

$$\boldsymbol{f}\left(\boldsymbol{z}(\varphi)\right) = \boldsymbol{f}\left(\psi(\boldsymbol{z})\right)\left|\psi(\boldsymbol{z})'\right| = \frac{1}{\pi\sqrt{\boldsymbol{L}^2-\boldsymbol{z}^2}}.$$

Otherwise:

$$\boldsymbol{f(x,z)} = \frac{1}{\sigma_x\sqrt{2\pi}}\frac{1}{\pi\sqrt{\boldsymbol{L}^2-\boldsymbol{z}^2}}\boldsymbol{e}^{-\frac{\boldsymbol{x}^2}{2\sigma_x^2}}.$$

Integral function can be determined as follows:

$$\boldsymbol{F(X)} = \int_{-\boldsymbol{L}}^{\boldsymbol{L}}\int_{0}^{\boldsymbol{X}-\boldsymbol{z}}\frac{1}{\sigma_x\pi\sqrt{2\pi\left(\boldsymbol{L}^2-\boldsymbol{z}^2\right)}}\cdot\boldsymbol{e}^{-\frac{\boldsymbol{x}^2}{2\sigma_x^2}}\boldsymbol{dz}\cdot\boldsymbol{dx} \Rightarrow$$

$$\boldsymbol{F(X)} = \frac{1}{\sigma_x\pi\sqrt{2\pi}}\int_{-\boldsymbol{L}}^{\boldsymbol{L}}\int_{0}^{\boldsymbol{X}-\boldsymbol{z}}\frac{1}{\sqrt{\left(\boldsymbol{L}^2-\boldsymbol{z}^2\right)}}\cdot\boldsymbol{e}^{-\frac{\boldsymbol{x}^2}{2\sigma_x^2}}\boldsymbol{dz}\cdot\boldsymbol{dx}.$$

Let us t change variable **z = L Cos(j).** Then:

$$dz = - L\ Sin(\varphi)\ d\varphi.$$

$$F(X) = \frac{1}{\sigma_x \pi\sqrt{2\pi}} \int_{-\pi}^{0}\int_{0}^{X-z} \frac{1}{\sqrt{\left(L^2 - L^2 Cos^2\varphi\right)}} \cdot e^{-\frac{x^2}{2\sigma_x^2}} \left(- L \cdot Sin\varphi \cdot d\varphi\right) \cdot dx.$$

Or

$$F(X) = \frac{1}{\sigma_x \pi\sqrt{2\pi}} \int_{-\pi}^{0}\int_{0}^{X-z} e^{-\frac{x^2}{2\sigma_x^2}} dz \cdot dx.$$

Element of the integral has no primitive function. For this case let it be presented as Taylor's series:

$$e^{-\frac{x^2}{2\sigma_x^2}} = 1 - a \cdot x^2 + \frac{\left(a \cdot x^2\right)^2}{2!} - \frac{\left(a \cdot x^2\right)^3}{3!} + \frac{\left(a \cdot x^2\right)^4}{4!} - \ldots, \quad \text{where } a = \frac{1}{2\sigma_x^2};$$

Or

$$e^{-\frac{x^2}{2\sigma_x^2}} = 1 - a \cdot x^2 + \frac{a^2 x^4}{2!} - \frac{a^3 x^6}{3!} + \frac{a^4 x^8}{4!} - \ldots,$$

Let us take an integral from this series as follows:

$$\int_{0}^{X-z} e^{-\frac{x^2}{2\sigma_x^2}} = x\Big|_0^{X1} - \frac{1}{3} a \cdot x^3\Big|_0^{X1} + \frac{1}{5}\frac{a^2 x^5}{2!}\Big|_0^{X1} - \frac{1}{7}\frac{a^3 x^7}{3!}\Big|_0^{X1} + \frac{1}{9}\frac{a^4 x^9}{4!}\Big|_0^{X1} - \ldots, \quad \text{where}$$

$x_1 = X - z.$

f(X) can be determined as follows:

$$f(x) = \frac{d \cdot F(X)}{dX}.$$

Then

$$f(X) = \frac{1}{\sigma_x \pi\sqrt{2\pi}} \int_{0}^{\pi} \frac{dx_1}{dx} \cdot \left[x\Big|_0^{X1} - \frac{1}{3} a \cdot x^3\Big|_0^{X1} + \frac{1}{5}\frac{a^2}{2!} x^5\Big|_0^{X1} - \ldots \right] \cdot d\varphi \Rightarrow$$

$$f(X)=\frac{1}{\sigma_x\pi\sqrt{2\pi}}\int_0^{\pi}e^{-a\cdot x_1^2}d\varphi=\frac{1}{\sigma_x\pi\sqrt{2\pi}}e^{-\frac{x^2}{2\sigma_x^2}}\int_0^{\pi}e^{\frac{2x\cdot L\cdot Cos\varphi-L^2Cos^2\varphi}{2\sigma_x^2}}d\varphi. \qquad (1)$$

Let us then denote **L = V$_c$ t .**

The equation (1) describes a density distribution of target's coordinates during some time after the moment when contact with the Target was lost. For 2D distribution on the plane it is possible to obtain an equation:

$$f(x,y)=\frac{1}{2\sigma_y\sigma_x\pi^3}e^{-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}}\int_0^{\pi}e^{\frac{2x\cdot L\cdot Cos\varphi-L^2Cos^2\varphi}{2\sigma_x^2}}d\varphi\cdot\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}e^{\frac{2y\cdot L\cdot Sin\varphi+L^2Sin^2\varphi}{2\sigma_y^2}}d\varphi.$$

The theorem is proven.

As related to the up-to-date computer technologies this report is prepared to be easily used for computer modeling, though there are some questions in this regard. The first question is a selection of an appropriate computer modeling technique. The second one is an interpretation of computer modeling results. In the above regard the computer interpretation of TSMO is a model of TSMO created by object-oriented approach.

From the theory of models point of view OOA application for computer modeling of TSMO is an interpretation of TSMO or the other model. In addition the next interpretation of TSMO, Fig. 7., is realized.

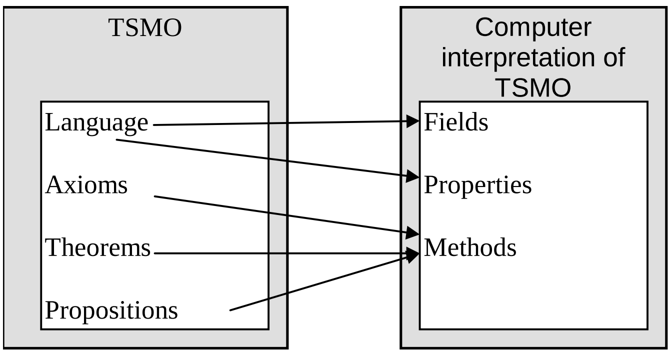


Fig.7. The version of object-oriented interpretation of TSMO.

Let us call the object-oriented interpretation of TSMO the object-oriented model of TSMO, or OOM.

The main procedures of OOM are: *incapsulation, inheritance, polymorphism*. They are shown in Fig.8.

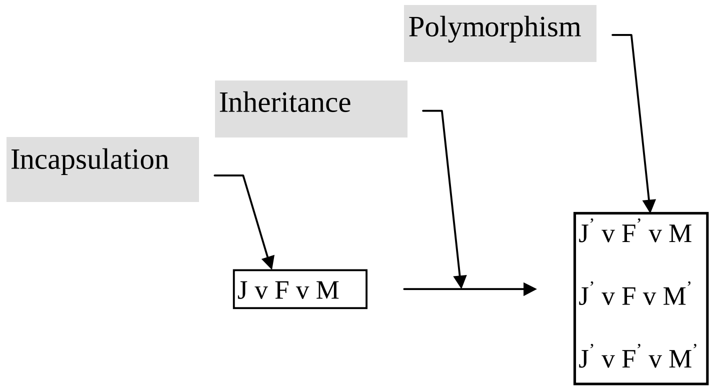


Fig.8. Procedures in the OOM.

Where:
- J – fields of object or class;
- F - properties of object or class;
- M – methods (functions, procedures) of object or class;
- V – logical union;
- $J^{'}, F^{'}, M^{'}$ - «overflowed» fields, properties, methods.

The OOM in contrast to TSMO is a formal axiomatic theory.

It should be noted that the theory of models is not a bad “tool” of the formal algebra. The use of this theory does not require a development of any special algebraic system. At the same time there is a possibility to use some fundamental theoretical results. Some of them are listed below.

1. The Levengame theorem (1915г): If a certain proposition has an endless model then the proposition has a counting model too.

In the beginning of this report it was shown that TSMO is the endless theory. By this theorem it can be proved that such a theory has at least one counting model. This is why it is correct to indicate TSMO as a consequence of the final counting models developed at PC.

2. The Hedel (1930г) - **Malthev** (1936) **theorem** (*theorem of compactness*): if each finite subset $\mathbf{X}_i$ of set **M** has a model then the set **M** has a model too.

In this case if a certain meta-class or process is a set of objects or classes, and they all are some models of applied field, then the meta-class or the process is a model of applied field too. On the other hand even if one object is not a model then the meta-class or the process cannot be considered a model of the applied field .

3. The Hedel theorem about incompleteness (1931г): any axiomatic theory included in the actual arithmetic (explicitly or implicitly) is incomplete.

The incompleteness is a fact that every proposition of some theory cannot be proven via the theory.

For our research this means that it is impossible to propose a special theory for automatic development for the models of search, e.g., the D. Gilbert’s [44] project. He wanted to build the consistent theory of numbers.

2. Development of the theorems of additivity and multiplicity

2.1. General aspects.

The advanced Observation System is a complex system. It contains many elements and subsystems. In real environment an Observer with a simple Observation System is rarely met. If we look at the animals' world we can see that each animal has more than one organ for observation. There are at least eyes for vision, ears for hearing and nose for a sense of smell. Different animals have different order of using their organs. Nevertheless they are using their organs together. Advanced ships, planes, helicopters and so on have different Observation Systems, and they are complex too, and they determine changes in different physical fields. The limitation of the classical TS against a circular observer zone is a very deep abstraction and does match practice.

The well-known theorem of additivity is solved as a part of this problem. In this regard this is a problem how to evaluate an effectiveness of the complex Observation System that includes subsystems for different fields of the Target detection. Currently we know a set of Observation Systems for different fields of the Target detection, such as optical, sonar, radio, electromagnetic, seismic and other. Some of them can determine a track of the Target, that may be a gas track, a heat track, a turbulent track and so on. Those can be on the water, in the air and on the land, of course.

These many types of the Observation Systems show that the problem is not a simple one. The direct solution will be not a good solution from the different points of view (time, money and so on). In [25] it was shown that the set of the Observation System, independent of the physical field, can be divided in two sets:

- Observation Systems that have the Observation zone like a circle or there exists a possibility to calculate the Observation zone as a circle or a sector;
- Observation Systems that have the Observation zone like a line or a rectangle or there exists a possibility to calculate the Observation zone like a line or a rectangle;

So, this complex problem will be formulated as a problem how to evaluate an effectiveness of two Observation Systems having different observation zones (circle and line) within one carrier. Let the Observe System with round zone be OS1 and the Observe System with linear zone OS2.

Let us discuss two hypothesizes as follows.

1.Let OS1 and OS2 be independent of each other during the search process. This means that the Target detection by one OS is independent of the Target detection by the other OS. Let this hypothesis be the additivity hypothesis in a wide sense (in comparison with the classical problem).

2. Let OS1 and OS2 depend on each other during the search process. This means that the process of the Target detection by one OS depends on the fact of the Target detection by the other OS. Let this hypothesis be the multiplicity hypothesis.

The example of the first hypothesis has been discussed in [28]. A peculiarity of this problem is a need to calculate not the common search potential but to distinct them because they are within one carrier. And in reality the common potential is always less than their regular sum.

One simple example can help to understand this problem. Let a carrier have two OS (OS1 and OS2). Evidently not the whole length of the OS2 zone will take part in the real search but only the length part $\mathbf{D_s}$ **-** $\mathbf{D_n}$, (see Fig.9.), where:
$\mathbf{D_s}$ **–** length of the Target's track (OS2);
$\mathbf{D_n}$ **-** radius of the zone (OS1).

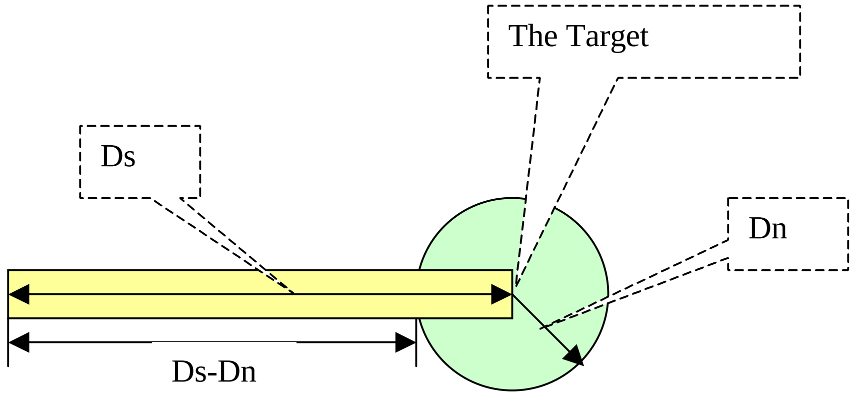


Fig.9. The additivity hypothesis.

Some authors have used this fact. But we do not know the full solution yet.

This problem's definition covers one more subproblem. There is a big difference between physical fields OS1 and OS2. According to this the detection of the Target is a real fact when the detection by OS2 was confirmed by OS1, and this depends on the real task. According to this remark we must study two variants of this problem as follows.

1). There is a detection of any physical fields of the Target (OS1 or OS2).

2). There is a detection of the Target only by OS1. This can be done either at once by OS1 or after detection by OS2

The problems of additivity and multiplicity have been formulated above. Let us discuss them in more detail.

2.2. Common theorem of additivity.

The well-known theorem of additivity is solved as a part of this problem. For the first time it was proposed by Koopman [24]. Let us formulate the task of Koopman as a theorem.

Theorem of additivity. *Search potentials of N (N>=1) independent observers have a property of additivity. The effectiveness of the Target search by Observers in the Region of search can be evaluated as:*

$$P(t) = 1 - e^{-\sum_{i=1}^{N} F_i}, \tag{2}$$

where **F**i = the search potential of i-Observer.

Equation (2) can be used only for one-type of Observation Systems. In the considered case this is a problem of evaluating an effectiveness of the complex Observation System that includes subsystems for different fields of the Target detection.

In comparison with the theorem by Koopman this task is common. Some limiting tasks were formulated and solved. These results showed an essence of the proposed approach as follows:

Theorem 1. *Let an Observer have two Observation Systems (OS1 & OS2). Let the Observer be looking for the Target. If Range of OS1 is limited by* $\boldsymbol{D_n}$ *, and OS2 is limited by* $\boldsymbol{D_s}$*, it is possible to determine conditions affecting the effectiveness of Observer's search.*

$$V_n > V_c \frac{D_n}{D_s}$$

Proof.

Let us see Fig.10. Let us believe the target is in the point O, its velocity $\mathbf{V_c} > \mathbf{V_n}$. Let us believe that meaning of effective width OS1 of Observer's track is - $\mathbf{D_n}$, length of target's track is - $\mathbf{D_s}$. It is needed to determine the Observer's velocity when OS*2* starts to influence upon a search potential of Observer.

From point O to circle D_n a tangent line is drawn. Parallelly to line DO a line from point C (the end of vector $\mathbf{V_c}$) is drawn.

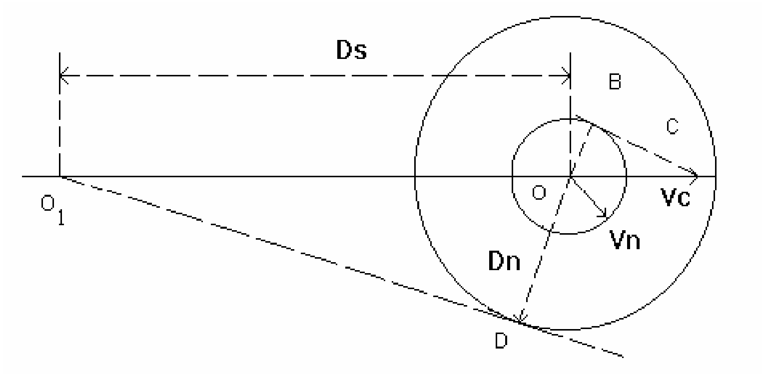


Fig.10. Observer velocity definition for case when OS2 influences upon common search potential of Observer.

Line approximation of Target's track.

From point O tangent to CB a circle is drawn. Length OB⊥CB.
From Δ **OBC** :

$$V_n = V_c Sin\alpha \tag{3}$$

From Δ O_1OD:

$$Sin\alpha = \frac{D_n}{D_s} \tag{4}$$

If we substitute (4) in (2) find the following:

$$V_n > V_c \frac{D_n}{D_s} \tag{5}$$

The theorem is proved.

Finally it is possible to denote the effect of OS2 using it only when V_n is in accord with a proviso (4).

Theorem 2. *Let us believe that the target's track is not a line but a rectangle. For this case it is possible to determine conditions that affect the effectiveness of Observer's search too.*

$$V_n \geq \frac{b+\sqrt{b^2-4ac}}{2a},$$

where

$a = D_s^2 + h^2;$
$b = 2\, V_c\, D_n\, D_s;$
$c = V_c^2(D_n - h^2).$

Proof.

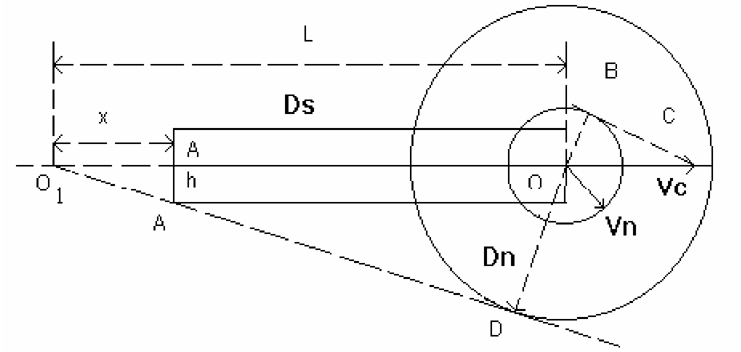


Fig. 11. Observer velocity definition for a case when OS2 influences upon common search potential of Observer. A case for rectangle approximation of the Target's track.

Similarly to the first theorem it is possible to determine Observer's velocity when OS2 influences upon common search potential of Observer. Let: **h** – is a half of the track's width. Tangent line to $\mathbf{D_s}$ through point A to cross a line of the track is drawn.

Δ **O_1AA**:

$$V_n = V_c Sin\alpha; \qquad \frac{D_n}{(D_s + x)} = Sin\alpha; \qquad \frac{h}{x} = tg\alpha \Rightarrow x = hCtg\alpha$$

Δ **OO_1D**:

$$V_n = V_c \frac{D_n}{(D_s + x)} = V_c \frac{D_n}{(D_s + hCtg\alpha)} \Rightarrow V_n = V_c \frac{D_n}{(D_s + hCtg\alpha)}$$

$$V_n = V_c \frac{D_n}{\left(D_s + h\dfrac{V_c}{V_n}Cos\alpha\right)} \Rightarrow V_n\left(D_s + h\frac{V_c}{V_n}Cos\alpha\right) = V_c D_n$$

$$V_n D_s + hV_c Cos\alpha = V_c D_n \Rightarrow hV_c Cos\alpha = V_c D_n - V_n D_s$$

$$hV_c\sqrt{1-\frac{V_n^2}{V_c^2}} = V_c D_n - V_n D_s \Rightarrow h\sqrt{V_c^2 - V_n^2} = V_c D_n - V_n D_s$$

Finally:

$$\mathbf{V_n^2(D_s^2 + h^2) - 2\,V_c\,V_n\,D_n\,D_s + V_c^2(D_n^2 - h^2) = 0.}$$

Let be:

$$\mathbf{a = D_s^2 + h^2;\quad b = 2\,V_c\,D_n\,D_s;\quad c = V_c^2(D_n^2 - h^2)\,.}$$

Then:

$$\mathbf{a\,V_n^2 - b\,V_n + c = 0;}$$

Solution of the above equation is:

$$\mathbf{V_{n_{1,2}} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}};\quad \Rightarrow \mathbf{V_n} \geq \frac{\mathbf{b + \sqrt{b^2 - 4ac}}}{\mathbf{2a}}\ .$$

The theorem is proved.

Theorem 3. *If conditions are the same as those in the theorems 1,2, it is possible to determine a part of target's track which determine an effectiveness of OS2 for the observer.*

a). $\mathbf{V_n <= V_c}$.

$$y = D_s - D_n\frac{V_c}{V_n} + hCtg(Sin^{-1}(\frac{V_n}{V_c}))$$

b). $\mathbf{V_n > V_c}$.

$$y = D_s - D_n.$$

Proof.

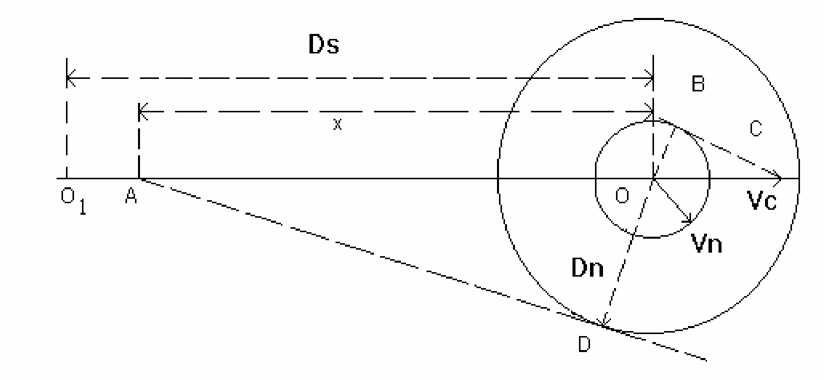


Fig.12. Effective length of the track.

Let us consider Fig.12. From point C tangent to the circle with radius $\mathbf{V_n}$ a line is drawn. Parallelly to this line a line tangent to the circle with radius $\mathbf{D_n}$.is drawn. Point **A** is obtained. It easy to see in Fig.12 that the Observer cannot cross the track in interval AO because in this case it must cross an area $\mathbf{D_n}$. An interval AO_1 is an effective length of the track. Let us determine a meaning of the interval.

a). V_n <= V_c.

Δ OBC, AOD

$$x = \frac{D_n}{Sin\alpha}\ , \qquad Sin\alpha = \frac{V_n}{V_c}; \qquad x = D_n\frac{V_c}{V_n}; \Rightarrow y = D_s - D_n\frac{V_c}{V_n} \tag{6}$$

Equation (6) is true only for linear approximation of the track. If we mean a rectangle approximation of the track an equation for effective length of the tract can be determined as follows:

$$\boldsymbol{y = D_s - D_n\frac{V_c}{V_n} + hCtg(Sin^{-1}(\frac{V_n}{V_c}))}\ . \tag{7}$$

The equation (7) is evident from Fig.13 .

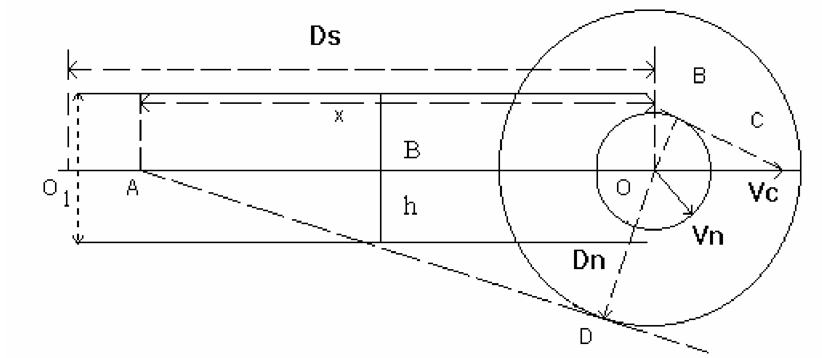


Fig.13. Effective length of the track. A case for rectangle approximation.

b).V_n > V_c.

For this case it is possible to write a simple equation as follows: $\mathbf{y = D_s - D_n}$.

Finally, for calculation of search effectiveness under additivity hypothesis for OS1 and OS2 it is better to use equations that were proved by this theorem. It should be noted that search potential of OS2 has to be determined under conditions a) and b).

The theorem is proved.

Theorem 4. *If there is a random search and the observation zone is a sector, it is possible to determine an effective projection of width of observer's search track.*

$$\overline{W} = \overline{W_1} \cdot P_2 + 2D_n P_1;$$

Proof.

Let us believe that an observation sector has an angle α. In this case, if a density of a Target distribution by Observer's bearing is known, it is possible to determine probability of Target appearance in some sector and beyond the sector.

Let be:
P_1 – probability of Targets appearance in the sector α **;**
P_2 – probability of Targets appearance beyond the sector α **;**

Then a common projection of Observer's search track can be determined as follows:

$$\overline{\boldsymbol{W}} = \overline{\boldsymbol{W}_1} \cdot \boldsymbol{P}_2 + 2\boldsymbol{D}_n \boldsymbol{P}_1; \qquad \text{where:}$$

$2D_n$ – radius of observation sector.

The theorem is proved.

Consequence 1*. For the case when* ***α≥π/2,*** *analytical equations for P_1 and P_2 can be obtained.*

$$P_1 = 2\frac{\alpha}{\pi} - 1, \qquad P_2 = 2\left(1 - \frac{\alpha}{\pi}\right), \quad \text{and} \qquad \overline{W'} = \frac{D_n}{\pi - \alpha} Sin\alpha.$$

Proof.

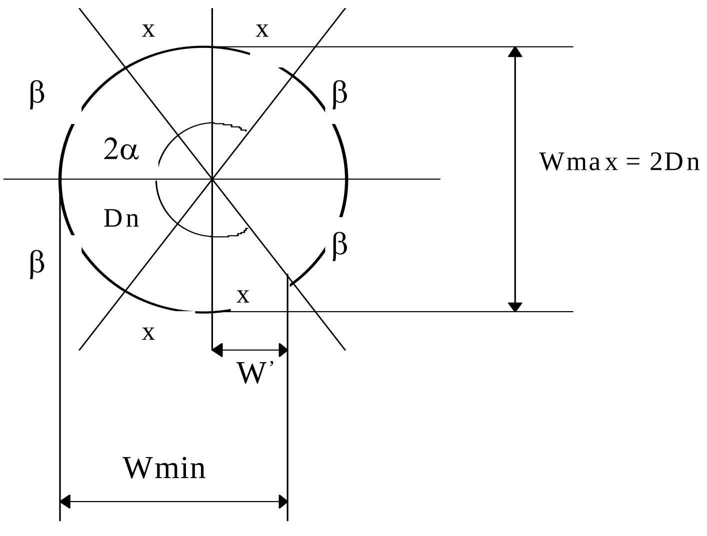


Fig.14.An effective width of observation zone.

A sector for $\mathbf{W_{max}}$ can be determined as follows:

$$\mathbf{X = 4x;\ x = \alpha - \pi/2;\ \longrightarrow X = 4\alpha - 2\pi}.$$

A probability of Target appearance in the sector for uniform distribution on the perimeter of $\mathbf{D_n}$ area (a case when Observer is stationary) can be determined as follows:

$$P_1 = \frac{4\alpha - 2\pi}{2\pi} = 2\frac{\alpha}{\pi} - 1$$

A sector for **W < W$_{max}$** can be determined as follows:

$$y = 2\pi - X = 2\pi - 4\alpha + 2\pi = 4(\pi - \alpha); \qquad P_2 = \frac{4(\pi - \alpha)}{2\pi} = 2\left(1 - \frac{\alpha}{\pi}\right)$$

So: $\quad . \; P_1 = 2\frac{\alpha}{\pi} - 1, \qquad P_2 = 2\left(1 - \frac{\alpha}{\pi}\right)$.

The common projection can be determined as follows :

$$\overline{W} = \overline{W_1} \cdot P_2 + 2D_n P_1; \qquad \text{where:}$$

$$\overline{W_1} = D_n + \overline{W'}; \qquad \overline{W'} = \frac{1}{\beta}\int_0^{\beta} D_n Cos\beta \cdot d\beta.$$

After solving the integral it is possible to obtain: $\overline{W'} = \frac{D_n}{\beta} Sin\beta;$. Let us make the remark that

$$\beta = \pi - \alpha.$$

Then $$\overline{W'} = \frac{D_n}{\pi - \alpha} Sin\alpha.$$

Finally, equation for the common projection is:

$$\overline{W} = D_n \frac{1 + Sin\alpha}{\pi - \alpha} P_2 + 2D_n P_1; \qquad \alpha < \pi .$$

The consequence is proved.

Consequence 2*. For the case* ***α≥π/2*** *and if a target is stationary an equation for the effective projection of width of observer's search track is*: $\overline{W} = 2D_n$.

Proof.

For this case: P_2 =0, and P_1 =1, because V_c =0 by definition. From the equation:

$$\overline{W} = D_n \frac{1 + Sin\alpha}{\pi - \alpha} P_2 + 2D_n P_1$$

it is possible to obtain:

$$\overline{W} = 2D_n$$

The consequence is proved.

Consequence 3. *For the case* $\alpha \geq \pi/2$ *and when* $V_n > 0$, $V_c > V_n$ *an equation for the effective projection of width of observer's search track can be determined as an equation.*

Proof.

A common equation will be the same: $\overline{W} = \overline{W_1} \cdot P_2 + 2D_n P_1$;

It is needed to determine P_1, P_2, because they depend on the density of Targets distribution by Observer's bearings. The density is a function of the arguments V_n / V_c.

Finally,

$$P_1 = 2\left(\int_0^x f(q)\cdot dq + \int_{\pi-x}^{\pi} f(q)\cdot dq\right); \qquad P_2 = 2\int_x^{\pi-x} f(q)\cdot dq.$$

The consequence is proved.

Consequence 4. *For the case* $\alpha \geq \pi/2$ *and when* $V_n >= V_c$ *and* $V_c > 0$ *an equation for the effective projection of width of observer's search track can be determined as an equation.*

Proof.

For this case:

$$P_1 = 2\left(\int_0^x f(q)\cdot dq + P_1'\right); \qquad P_1' = 2\left(\int_{\pi-x}^{\pi-\theta} f(q)\cdot dq\right); \quad \theta = \arcsin\left(\frac{V_c}{V_n}\right);$$

$$P_2 = 2\left(\int_x^{\pi-x} f(q)\cdot dq\right);$$

Condition: $\pi - \Theta > \pi - x \to \Theta > x \to \Theta > \alpha - \pi/2.$

For a case, when: $\Theta \leq \alpha - \pi/2 \to P_1 = 0.$

$$P_2 = 2\left(\int_x^{\pi-\theta} f(q)\cdot dq + P_1'\right);$$

If $\Theta \leq 2\alpha - \pi/2$, then $P_2 = 0$ and $W = 2Dn$.

The consequence is proved.

Theorem 5. *If during the random search the Observer can determine the Target track the effective projection of the Target track can be determined.*

$$\bar{z}_0 = \frac{4C}{\pi}\left(Cos(\varphi + \alpha) + Sin\varphi\right)$$

Proof.

Let us believe that a Target and an Observer are random moving in a Region. They have constant velocity V_n and V_c correspondingly. Let us believe that an intensity of Observer's search can be determined as follows:

$$\gamma = \frac{WV_\rho}{S}, \qquad \text{where:}$$

W - effective projection of search track;
S - square of search Region.
V_ρ - relative velocity of Target's moving.

For this theorem **W** is an average of distribution of rectangle length on a line that is a norm to Observer course. It is true when and only when an Observer detects Target's track when the cross angle belongs to the sector. Let the sector be (0, α). It is a shadow sector, Fig.15.

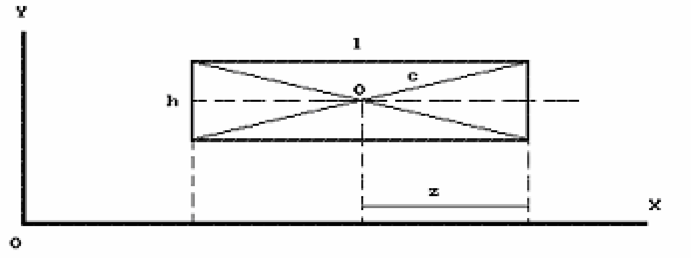


Рис.15. Effective projection of Target's track.

It is easy to see that there are four similar sectors in Fig.15 and it is also easy to see that the rectangle projection can be defined through its diagonals. The diagonal length equals **2C**. A meaning of **C** can be obtained as follows:

$$C = \frac{h}{2Sin\varphi} = \frac{l}{2Cos\varphi}, \qquad \text{where:}$$

φ - an auxiliary angle.
Z projection can be obtained as follows:

$$z = CCos(\varphi - \zeta), \qquad \text{where:}$$

ζ - an angle between a line of track and a norm to Observer's course. In Fig.15 this is an angle between ℓ and the datum line: OX.
$\zeta \in 0,2\pi$, because it is a case of random moving. It is easy to see, Fig.15, random moving of Target and Observer equals to rolling of rectangle along its own center of symmetry. Evidently, a meaning of the projection **z** during the rolling changes as follows:

$$\zeta = \left(0, \frac{\pi}{2}\right)$$

Let us obtain an average of distribution of **z** . Let us denote that:

$$\zeta \in \left(0, \frac{\pi}{2} - \alpha\right), \text{ where:}$$

α - the quarter of a shadow sector. An average of **z** can be obtained as average on the interval as follows:

$$\bar{z}_1 = \frac{2}{\pi - 2\alpha}\int_0^{\frac{\pi}{2}-\alpha} CCos(\varphi - \zeta)d\zeta$$

Let us make a substitution: $t = \varphi - \zeta; \qquad d\zeta = -dt.$ Then:

$$\bar{z}_1 = -\frac{2}{\pi - 2\alpha}\int_{\varphi}^{\varphi+\alpha-\frac{\pi}{2}} CCostdt = -\frac{2}{\pi - 2\alpha} S\text{int}\Big|_{\varphi}^{\varphi+\alpha-\frac{\pi}{2}} = -\frac{2C}{\pi - 2\alpha}(Cos(\varphi+\alpha) + Sin\varphi).$$

$\bar{z}_1$ has been obtained for $\zeta \in \left(0, \frac{\pi}{2} - \alpha\right)$. For the interval: $\zeta \in (0, 2\pi)$ it is easy to write:

$\bar{z} = \bar{z}_1 P_1 + 0P_2$, where:

$$P_1 = \frac{2\pi - 4\alpha}{2\pi} = 1 - \frac{2\alpha}{\pi};$$

P_1 – a probability that Observer can be in the sector where a Target track detection is possible;

$$P_2 = \frac{4\alpha}{2\pi} = \frac{2\alpha}{\pi} \ ;$$

P_2 - a probability that Observer can be in the shadow.
In this sector Target can not be detected by definition, so: $\bar{z} = \bar{z}_1 P_1$. and then:

$$\bar{z} = \frac{\pi - 2\alpha}{\pi}\frac{2C}{\pi - 2\alpha}\big(Cos(\varphi+\alpha) + Sin\varphi\big) = \frac{2C}{\pi}\big(Cos(\varphi+\alpha) + Sin\varphi\big).$$

The common projection will be two times more because: $\bar{z}_0 = 2\bar{z}$.

Finally it is easy to obtain a common equation for effective track projection as follows:

$$\bar{z}_0 = \frac{4C}{\pi}\big(Cos(\varphi+\alpha) + Sin\varphi\big)$$

The theorem is proved.

So, at this phase a common case of the theorem of additivity was examined. The theorem of additivity by Koopman was extended to some actual cases such as:

- The case, when the Observation zone is approximated by a circle, or it is possible to calculate the Observation zone as a circle or a sector;
- The case, when the Observation zone is approximated by a line or a rectangle, or it is possible to calculate the Observation zone as a line or a rectangle;
- The case, when OS1 and OS2 are independent of each other during the search process;
- The case, when OS1 and OS2 depend on each other during the search process.

The new results can be used directly to solve some practical problems, and computer modeling is the best way to use them. (see paragraph 3.)

2.3. *Theorem of multiplicity: the first and the second definitions.*

Conventionally, a complex system is meant under the advanced Observation System containing many elements and subsystems. It seldom happens that the Observer uses a simple Observation System to solve real tasks. Here a certain parallel could be made with the animal observation skills sharpened by a complex parallel or simultaneous use of various observation organs and functions (eyes-vision, ears-hearing, nose-smelling, brain-perceptions processing). To detect changes in a variety of physical fields modern carriers also operate with different complex Observation Systems. As a rule during the search process all information coming from different observation systems is used together. However, the classical theory of search does not give any solution for this problem. A solution for two different interdependent observation systems within one carrier is proposed. The Observation System with a round zone is called OS1, and the Observation System with a linear zone is called OS2.

For the first time the case, when observation systems are independent and uniform in the search process, was described by B.O. Koopman. The case considering independent and different observation systems will be considered now. It seems possible to present the following theorem.

Theorem 6. *Let OS1 and OS2 be within the Observer that is looking for the Target. Let OS1 and OS2 depend on each other during the search process. If a range of OS1 is determined as* $\boldsymbol{D_n}$ *and OS2 is determined as* $\boldsymbol{D_s}$, *and the Target's relative track does not cross the Observer's observation area (OS1), it is possible to determine computationally a probability area of target localisation (PATL) in case when the target's track is detected by OS2 and not detected by OS1 yet.*

A common schema of PATL construction can be as shown in Fig. 16.

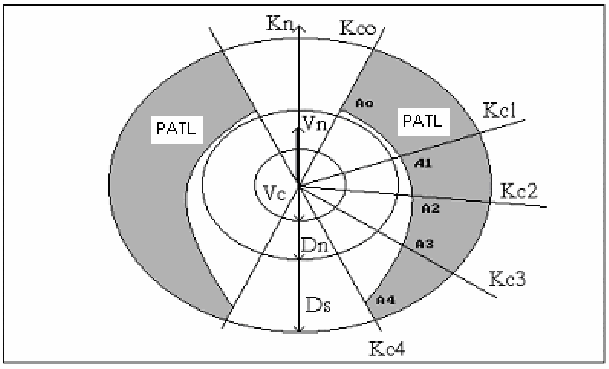


Fig.16. A common schema of PATL construction.

Where:

$$D_{\min i} = \frac{D_n V_{\rho i}}{V_n Sin K_{ci}}.$$

$$K_{c\min} = Sin^{-1}(b) - Sin^{-1}(a).$$

$$K_{c\max} = \pi - Sin^{-1}(b) - Sin^{-1}(a).$$

Proof.

Let there be an Observer operating with OS1 and OS2, and the Observer be in a Region. The Observer is looking for the Target. Let the Observer have a fixed course and a velocity. Evidently, the width of observer's search track for OS1 is D_{ef}. Let the Target localization inside

an observation area for OS1 be detected with probability close to 1. In this case it is possible to denote that the Target could be behind Observer's beam when the Target's track has been detected by OS2 and the Target could not been detected by OS1 when and only when the Target's line of relative moving has not crossed the observation area by OS1 because in this case it has been detected by OS1. As another variant it is possible to formulate the following statement: where is the Target detected by OS2 and not detected by OS1 located?

For a general case this situation is shown in the Fig.17. In this regard a problem of tactical maneuvering can be formulated as follows: calculate a sector of Target's possible courses when the Target crosses Observer's course in front of Observer and the Target is in the distance >= $\mathbf{D_{ef}}$. It is shown more details on the construction of the scheme based on a well-known triangle of velocities.

After that it is possible to make some comments.

1. If a velocity of the Target is not too high there is a reason to decrease the Observer's velocity (V_n) if other conditions are not contradicted (e.g., a time of search, a great value of square of search and so on).

2. If a velocity of the Target is rather high there is no reason to change Observer's velocity (V_n) because it does not influence upon a square of PATL ($S_{овмц}$).

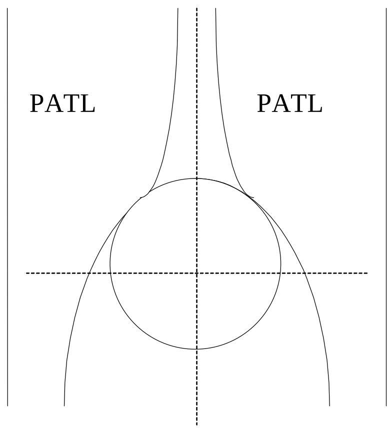


Fig.17. A common case of PATL.

According to the task let us believe:

$$\mathbf{D_{max}} = \mathbf{D_s} = \mathbf{V_c}t \quad \text{, where:}$$

$\mathbf{V_c}$ – velocity of Target;
$\mathbf{D_s}$ – length of Target's track.

Let us use a schema in Fig.18 to explain a principle of PATL construction. There is a well known triangle of velocities in Fig.18, where:

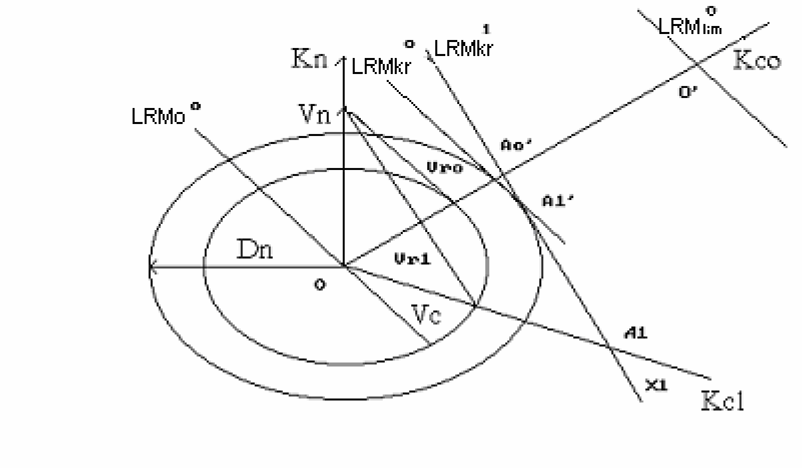


Fig.18. A principle of PATL construction.

V_n – vector of Observer's velocity;
V_c - vector of Target's velocity;
V_ρ - vector of relative moving.

$$\mathbf{V_\rho = V_n - V_c}.$$

Remark. ψ is an angle between **K_n** and a real meaning of Target course **K_{ci}** **K_{ci}** $\in (0.....\pi)$. in Fig.18.

Let us believe that the Observer is relative stationary in the point O. On the line of Target course let us mark a length **$OO' = D_s$** .

A set of lines relative moving (LRM) [LRMo,...,LRMkr] is a set of possible ways on the length OO' for the Target. In other words those are possible situations to cross Target course on astern of Target in the distance not more than **D_{ef}** . It easy to see, Fig.18, that a part of LRM crosses an area, limited by radius **D_{ef} .** A critical LRM that divides two groups LRM (some of them cross **D_{ef}** and some of them do not cross **D_{ef}**) is an LRMkr tangent to circle of **D_{ef}** radius.

It is needed to obtain a distance **OA_o** in the direction **K_{co}** . An **OA_o** length is a minimal distance where the Target can be if it is not in **D_{ef}** area. A maximum distance can be determined as a length of the track - **D_s** .

A problem to obtain the **OA_o** length can be solved in various ways.

1.An analytical solution.
Let us obtain a minimum distance of a Targets on the course.
Let us consider Fig.18. From $\Delta OA\mathbf{A_1}$ according to Cos theorem (accounting for a triangle of velocities) it is possible to write the following:

$$V_{\rho 1} = \sqrt{V_n^2 + V_c^2 - 2V_nV_cCosK_{c1}} \quad .$$

According to Sin theorem:

$$\frac{V_n}{SinX_1} = \frac{V_{\rho 1}}{SinK_{c1}} \Rightarrow X_1 = Sin^{-1}\left(\frac{V_n}{V_{\rho 1}} SinK_{c1}\right) \quad .$$

Let **$OA_1 = D_{min1}$** .

According to **Sin** theorem:

$$\frac{D_n}{D_{\min 1}} = SinX_1 \Rightarrow D_{\min 1} = \frac{D_n V_{\rho 1}}{V_n SinK_{c1}}.$$

The **D_s - D_{min1}** length for the **K_{c1}** is received. This is a length where the Target can be, and it has not crossed **D_{ef}** area.

For PATL construction it is possible to write a common equation as follows:

$$D_{\min i} = \frac{D_n V_{\rho i}}{V_n SinK_{ci}}.$$

For $\forall K_{ci}$ put Δ **$D_i = D_s - D_{mini}$** lengths from **D_{mini}** it is possible to receive a common PATL.

Evidently, see Fig.18, the PATL has real boundaries for the **D_s**. In this case it is possible to obtain **K_{cmin}** and **K_{cmax}** when **$D_{min} = D_s$** . And PATL will be limited by sector (**K_{cmin}, K_{cmax}**).

PATL is obtained. Let us discuss a problem how to decrease a square of PATL. It is needed for Target localization inside PATL. For PATL study let us solve two tasks:

a). Obtaining **K_{cmin}, K_{cmax}** .
b). Conditions definition that can decrease PATL square.
a). The first task. This task can be solved in two ways: analytical and graph - analytical.

Analytical way.

It is needed to solve an equation as follows:

$$D_s = \frac{D_n V_\rho}{V_n SinK} \ .$$

Let us rewrite this equation in differently:

$$\left(\frac{D_s V_n}{D_n}\right)^2 Sin^2 K = V_n^{\ 2} + V_c^{\ 2} - 2V_c V_n \ CosK \Rightarrow$$

$$\left(\frac{D_n}{D_s}\right)^2 + \left(\frac{V_c D_n}{D_s V_n}\right)^2 - 2\left(\frac{D_n}{D_s}\right)^2 \frac{V_n}{V_c} CosK = 1 - Cos^2 K \ .$$

Let

$$b = \left(\frac{D_n}{D_s}\right)^2; a = \left(\frac{D_n}{D_s m}\right)^2; m = \frac{V_n}{V_c} \ .$$

After substitution it is possible to receive the following equation:

$$Cos^2K - 2bmCosK + b + a - 1 = 0;$$

Let: **c = a + b - 1**, then:

$$Cos^2K - 2bmCosK + c = 0;$$

Let: $a' = bm, d = \sqrt{b^2m^2 - c}$,

Finally:

$$K_{cmax} = Cos^{-1}(a' - d);\ \ K_{cmax} = Cos^{-1}(a' + d).$$

Graph - analytical way.

Let us consider a case, see Fig.19, for K_{cmax} .

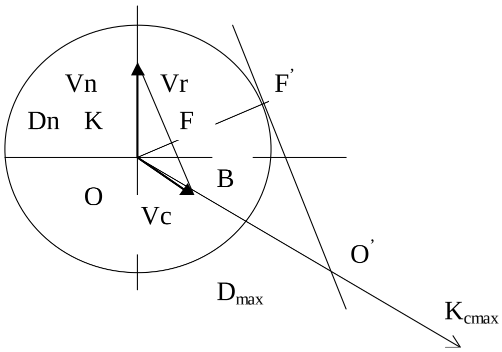


Fig.19. A case for maximum Target course.

Δ **OAB**. According to Sin theorem:

$$\frac{V_\rho}{SinK} = \frac{V_n}{SinX} \Rightarrow SinX = \frac{V_n}{V_\rho} SinK;$$

Δ **OFO**. According to Sin theorem:

$$\frac{D_n}{D_m} = SinX;$$

Let us denote $D_m = D_s$ by definition: $\frac{D_n}{D_s} = SinX;$

Then:

$$\frac{D_n}{D_s} = \frac{V_n}{V_\rho} SinK, \qquad \frac{OF}{V_c} = SinX \Rightarrow OF = V_c SinX; \text{ but } Cosy' = \frac{OF}{V_n},$$

Where:

x, y - auxiliary angles.

Comparison of two equations for OF allows to receive: $Cosy' = \frac{V_c}{V_n} SinX$.

Substitution of **Sin(x)** can help to receive:

$$Cosy' = \frac{V_c D_n}{V_n D_s} \Rightarrow y' = Cos^{-1}\left(\frac{V_c D_n}{V_n D_s}\right).$$

From Δ **OAB** evidently: $\mathbf{K_{cmax} = y + y'}$.

From Δ **OFO**:

$$y = \frac{\pi}{2} - x = \frac{\pi}{2} - Sin^{-1}\left(\frac{D_n}{D_s}\right).$$

Then:

$$K_{c\max} = Cos^{-1}\left(\frac{V_c D_n}{V_n D_s}\right) - Sin^{-1}\left(\frac{D_n}{D_s}\right) + \frac{\pi}{2}.$$

Next conditions must be accounted for:

$$\frac{D_n}{D_s} \le 1, \qquad \frac{V_c D_n}{V_n D_s} \le 1.$$

These conditions evidently belong to the last equation. A physical interpretation is: if $\mathbf{D_s < D_n}$, then OS2 does not influence upon the search potential of Observer.

Let us discuss a case, see Fig.20, for $\mathbf{K_{cmin}}$.

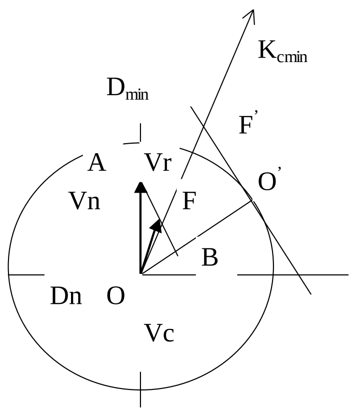


Fig.20. A case for minimum Target course.

Δ OF'O'. According to Sin theorem:

$$x = Sin^{-1}\left(\frac{D_n}{D_s}\right); \qquad \Delta OAB: \qquad \frac{OB}{V_n} = Sin\alpha, \qquad \frac{OB}{V_c} = Cosy.$$

So: $OB = V_c \dfrac{D_n}{D_s}$, or: $Sin\alpha = \dfrac{V_c D_n}{V_n D_s}$.

From Δ **OAF**: $y' + \alpha = x \Rightarrow y' = x - \alpha$, or:

$$y' = Sin^{-1}\left(\frac{D_n}{D_s}\right) - Sin^{-1}\left(\frac{V_c D_n}{V_n D_s}\right).$$

But by definition: $\mathbf{K_{cmin} = y'}$.

Then:

$$K_{c\min} = Sin^{-1}\left(\frac{D_n}{D_s}\right) - Sin^{-1}\left(\frac{V_c D_n}{V_n D_s}\right).$$

The conditions here are the same as for $\mathbf{K_{cmax}}$:

$$\frac{D_n}{D_s} \le 1, \qquad \frac{V_c D_n}{V_n D_s} \le 1.$$

Let $b = \left(\dfrac{D_n}{D_s}\right), \qquad a = \left(\dfrac{V_c D_n}{V_n D_s}\right);$

Finally it is possible to write:

$$K_{c\min} = Sin^{-1}(b) - Sin^{-1}(a). \quad K_{c\max} = \pi - Sin^{-1}(b) - Sin^{-1}(a).$$

b). This task belongs to task of determining a figure square between $K_{c\min}, K_{c\max}$ and finding how to decrease the square. PATL can be created as follows:

$$D_{\min i} = \frac{D_n V_{\rho i}}{V_n Sin K_{ci}}.$$

Evidently, $\mathbf{D_{mini}}$ is a function of $\mathbf{K_{ci}}$ variable. Let the function be like $\mathbf{D_m(K)}$. Let us use Cartesian system of coordinates. Graphics of functions $\mathbf{D_s(K)}$ and $\mathbf{D_m(K)}$ are shown in Fig.21. It is possible to see that $\mathbf{D_m(K)}$ is a piecewise smooth, differentiable on an interval with the exception of special points: $K_c = l\pi, \quad l = 0, \pm 1, \pm 2, \ldots$. In these points the function has discontinuity of the first kind (it exists in the neighborhood of points K_l, but does not exist exactly in the points).

Let us look at an interval $(0, \pi)$. It corresponds to the right side of Observer. It is easy to see, Fig.21, that a square of the figure under the curve $\mathbf{D_m(K)}$ can be calculated as follows:

$$S' = \int_{Kc\min}^{Kc\max} \frac{D_n V_\rho}{V_n SinK} dK.$$

In this case a square of PATL can be obtained as follows:

$$S = D_s^2 \Delta K - S', \quad \text{where:} \qquad (8)$$

$D_s^2 \Delta K$ - square of figure under D_s ;
$\Delta K = K_{c\max} - K_{c\min}$.

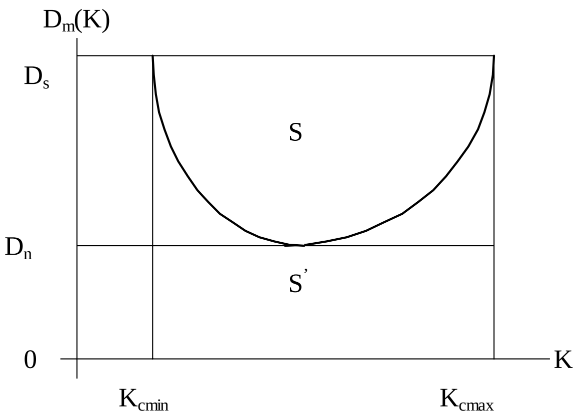


Fig.21. PATL on the flat.

It is needed to decrease PATL square. Let us obtain conditions for decreasing square of PATL. It is possible to see, the last equation, that the Observer can change ΔK and S'. Let us study conditions that increase S'. Let us write an equation for $\mathbf{D_m(K)}$ as follows:

$$D_m(K) = \frac{D_n(V_n, V_c) V_\rho (V_n, V_c, K)}{V_n SinK}.$$

One can see that the variables for $\mathbf{D_m(K)}$ are $\mathbf{V_n}$, $\mathbf{V_c}$, **K**. Observer can only influence upon one variable $\mathbf{V_n}$ that its own velocity. In this regard let us consider function $\mathbf{D_m(K)}$ as function of $\mathbf{V_n}$. Let us believe that other variables are constants. Functional dependence of $\mathbf{D_n}$ on $\mathbf{V_n}$ can be shown analytically and graphically using experimental data. For approximate calculation let us take the following law for $\mathbf{D_n(V_n)}$:

$$D_n(V_{n\min}^{\min}) = 0.9 D_0, \qquad D_n(V_{n\min}^{\max}) = 0.6 D_0, \qquad D_n(V_{n\max}) = 0, \quad \text{где:}$$

$\mathbf{D_0}$ - potential range of Observer.

For a practical illustration it is easy to take constant meaning of variables $t_j = Const, \quad V_c = Const.$ and to create graphics of $\mathbf{D_n(V_n)}$.

Results of computer modeling show the decreasing of PATL square; it is needed to decrease a velocity of Observer.

Let as prove these results for a common case.

Let us rewrite equation for **S’** in another way:

$$S' = \frac{\Delta K}{3n}\left(\frac{D_n V_{\rho 0}}{V_n Sin K_0} + 4\frac{D_n V_{\rho 1}}{V_n Sin K_1} + \ldots + \frac{D_n V_{\rho 2n}}{V_n Sin K_{2n}}\right) =$$

$$= \frac{\Delta K D_n}{3n V_n}\left(\frac{V_{\rho 0}}{Sin K_0} + 4\frac{V_{\rho 1}}{Sin K_1} + \ldots + \frac{V_{\rho 2n}}{Sin K_{2n}}\right).$$

Let

$$h' = \frac{\Delta K}{3n}; \qquad y_i = \left(\frac{V_{\rho i}}{Sin K_i}\right).$$

$$S_1' = y_0' + 4y_1' + 2y_2' + 4y_3' + \ldots + 2y_{2n-2}' + 4y_{2n-1}' + y_{2n}';$$

Then

$$S' = \frac{h' \cdot D_n}{V_n} S_1'.$$

It is needed to increase **S’** because in this regard PATL is decreasing automatically.
From the last equation for **S’** it is easy to see that for decreasing of PATL it is needed to perform the following:
a).To increase $\mathbf{D_n}$.
b).To increase $\mathbf{S'_1/V_n}$.

If $\mathbf{V_n}$ is increasing then $\mathbf{D_n}$ **---> 0,** and evidently **S’--->0** . So, for decreasing of **S** it is needed to decrease $\mathbf{V_n}$**.**

Let us believe that if $\mathbf{V_n}$ is increasing then $\mathbf{D_n}$ is not tending to zero, and is tending to its low bound: $\mathbf{inf}(D_n)$**.** For this case let us examine a proportion: $\frac{V_{\rho i}}{V_n}$**,** The proportion is a part of **S’** .

Let us write the proportion as follows:

$$\frac{V_\rho}{V_n} = \frac{\sqrt{V_n^2 + V_c^2 - 2V_c V_n Cos\theta}}{V_n}.$$

Results of computer modeling showed that for PATL decreasing it is needed to decrease $\mathbf{V_n}$. Then a meaning of $\mathbf{V_n}$ is lower than a meaning of S is lower too. It does not depend on:

$$D_n = \sup(D_0)\big|V_n = V_{\min}^{\min}.$$

Proof.
It is needed to examine a proportion as follows (for a case when $\exists\, \mathbf{inf}(D_0)$):

$$\lim_{Vn\to\infty}\frac{V_\rho}{V_n}.$$

For a common case a solving of this bound is not an easy task. Let us use boundary conditions as follows:

a). $\theta = 0$; b). $\theta = \frac{\pi}{2}$.

Minimum and maximum meanings of V_ρ correspond to these conditions as follows:

a). $$\lim_{Vn\to\infty} \frac{V_n - V_c}{V_n} = \lim_{Vn\to\infty} \left(1 - \frac{V_c}{V_n}\right) = 1.$$

b). $$\lim_{Vn\to\infty} \frac{\sqrt{V_n^2 + V_c^2}}{V_n} = \lim_{Vn\to\infty} \sqrt{1 - \frac{V_c^2}{V_n^2}} = 1.$$

It is possible to make a conclusion: for a common case the proportion $\frac{V_{\rho i}}{V_n}$, tends to 1 if V_n is increasing.

Remark. Let us show that if V_n is increasing then $\Delta K = K_{c\max} - K_{c\min}$. is increasing too. There are two variants for this case as follows:

a). $\inf(D_n) \to 0$, if $\mathbf{V_n>=V_{max}}$.

Let us write equation for $\mathbf{K_{cmin}}$, $\mathbf{K_{cmax}}$:

$$K_{c\min} = Sin^{-1}\left(\frac{D_n}{D_s}\right) - Sin^{-1}\left(\frac{V_c D_n}{V_n D_s}\right). \quad K_{c\max} = \pi - Sin^{-1}\left(\frac{V_c D_n}{V_n D_s}\right) - Sin^{-1}\left(\frac{D_n}{D_s}\right).$$

Evidently, for this case: $K_{c\max} \to \pi, \qquad K_{c\min} \to 0 \Rightarrow \Delta K \to \pi.$

b). $\inf(D_n) > 0$

From equations $\mathbf{K_{cmin}}$, $\mathbf{K_{cmax}}$ it is easy to see: $K_{c\max} \to \pi, \qquad K_{c\min} \to 0 \Rightarrow \Delta K \to \pi.$

Conclusion.

1. Decreasing of $\mathbf{V_n}$ influence upon decreasing of PATL.
2. If $\mathbf{t_j}$ is increasing, $\mathbf{V_n}$ does not influence upon $\mathbf{S_{овмц}}$.
3. For a search of slow Target it is possible to decrease $\mathbf{V_n}$ for $\mathbf{S_{овмц}}$ decreasing. It can be done if it does not contradict woth other conditions (e.g., a short search time, a large search region, etc.).
4. For a case of fast Target $\mathbf{V_n}$ does not influence upon $\mathbf{S_{овмц}}$.

It has been noticed that the first definition of the theorem of multiplicity is correct only for some practical cases. It is so because it was proposed that the Target's relative track does not cross the Observer's observation's area (OS1). But some times it is not exactly so. In a broad sense the second definition of the theorem of multiplicity is proposed.

Theorem of multiplicity (the second definition). *Let there be OS1 and OS2 within the Observer that is looking for the Target. Let OS1 and OS2 depend on each other during the search*

process. If a range of OS1 is determined as $\boldsymbol{D_n}$ *and OS2 is determined as* $\boldsymbol{D_s}$*,, it is possible to determine computationally a probability area of target localisation (PATL) in case when the target's track is detected by OS2 but the Target not detected by OS1 yet.*

The PATL can be determined as follows:

$$S' = \int_{Kc\min}^{Kc\max} D_x dK \,,$$

where:

$$D_x = S_\rho Cos\phi + \sqrt{D_n^2 - S_\rho^2 \cdot Sin^2\phi} \,,$$

where:

$$\phi = \alpha + K_{ci}; \qquad \alpha = Sin^{-1}\left(\frac{V_c}{V_\rho} SinV_c\right) \quad ,$$

where:

$$\boldsymbol{S_\rho = V_\rho \cdot t_n}$$

Proof.

It is needed to determine a possibility that Target is on the course if the Target is not been in the Observation area of Observer, see Fig.22.

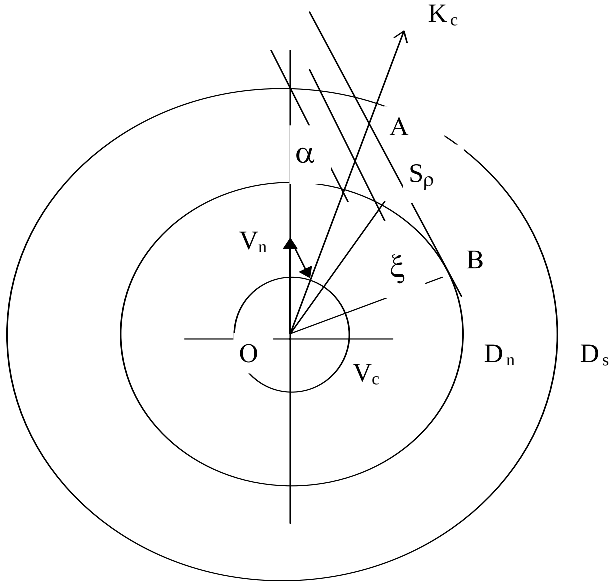


Fig. 22. Target's lines of relative moving. A case for $\boldsymbol{V_n > V_c}$.

In the very beginning of our proof let us determine conditions for LRM to cross $\mathbf{D_n}$ zone. From Δ **OAB** let us obtain a distance on the line of course K_{ci} as follows:

$$\boldsymbol{OA = \frac{D_n}{Sin\alpha}; \qquad Sin\alpha = \frac{V_c}{V_\rho} SinK_{ci}}.$$

If $\mathbf{OA < D_s}$, not all LRM will cross $\mathbf{D_n}$ zone.

For this case it is possible to obtain a maximum meaning of relative moving of Target before **D$_n$**, zone. In other words it is possible to determine a fact of a Target presence possibility, its track has been detected, in the **D$_n$** zone.

Let $$\frac{D_n}{Sin\alpha} = \frac{D_s}{Sin\xi} \Rightarrow Sin\xi = \frac{D_s}{D_n} Sin\alpha;$$

And $$\frac{S_\rho}{Sin(\pi - (\alpha + \xi))} = \frac{D_n}{Sin\alpha}; \Rightarrow S_\rho = \frac{D_n}{Sin\alpha} Sin(\alpha + \xi),$$ where:

$$\alpha = Sin^{-1}\left(\frac{V_c}{V_\rho} SinK_{ci}\right); \qquad \xi = Sin^{-1}\left(\frac{D_s}{D_n} Sin\alpha\right) = Sin^{-1}\left(\frac{D_s V_c}{D_n V_\rho} SinK_{ci}\right).$$

In this case it is possible to determine a time that needed for Target to pass a distance S_ρ.

$$t_\rho = \frac{S_\rho}{V_\rho}.$$

For detection of delay time for OS2 let us see Fig.23.

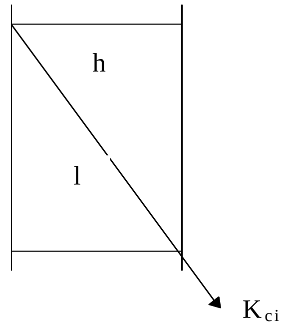


Fig.23. A case for track crossing.

Let us believe that the OS2 is working in real time. If it is not so it is needed to increase delay time at least at the time of decision support **t$_k$**.

$$l = \frac{h}{SinK_{ci}} \Rightarrow t_n = \frac{h}{V_n SinK_{ci}}.$$

A condition that the Target is on the course can be written as follows:

$$t_\rho > t_n + t_k.$$

Some time it easy to solve the inverse task (for graphical representation).
Let us obtain a point on the course of Target – on the line of K$_{ci}$. Starting at this point on the course there might be a target, whose track was crossed by Observer.
Let us write: $S_\rho = V_\rho \cdot t_n$. According to **Cos** theorem:

$$D_x^2 + S_\rho^2 - 2 \cdot D_x S_\rho \cdot Cos\phi = D_n^2; \Rightarrow D_x^2 - 2 \cdot D_x S_\rho \cdot Cos\phi + S_\rho^2 + D_n^2 = 0;$$

Where:

$$\phi = \alpha + K_{ci}; \qquad \alpha = Sin^{-1}\left(\frac{V_c}{V_\rho} SinV_c\right).$$

Then:

$$D_x = S_\rho Cos\phi + \sqrt{D_n^2 - S_\rho^2 \cdot Sin^2\phi}.$$

Interval of Target if it is on the course must match the following condition: **D_x<=D_s**.
If $D_x < D_{min}$, for calculation it is needed to use D_x.
if $D_x \geq D_{min}$, for calculation it is need to use D_{min}.
Finally, the integrand equation is calculated differently than in the previous theorem (the first definition). The equation can be like that:

$$S' = \int_{Kc\,min}^{Kc\,max} D_x dK.$$

3. Development of the computer prototype.

3.1.Development technology.

According to the OOA the computer prototype for our research has been developed. This is a reasonable rule to start a computer modeling in the beginning of the applied field analysis. The computer prototype (CP) was developed for the previous computer modeling. The CP is based on the special technology proposed for this research, see Fig.24. The architecture of CP is shown in Fig.25.

Proposed Technology.

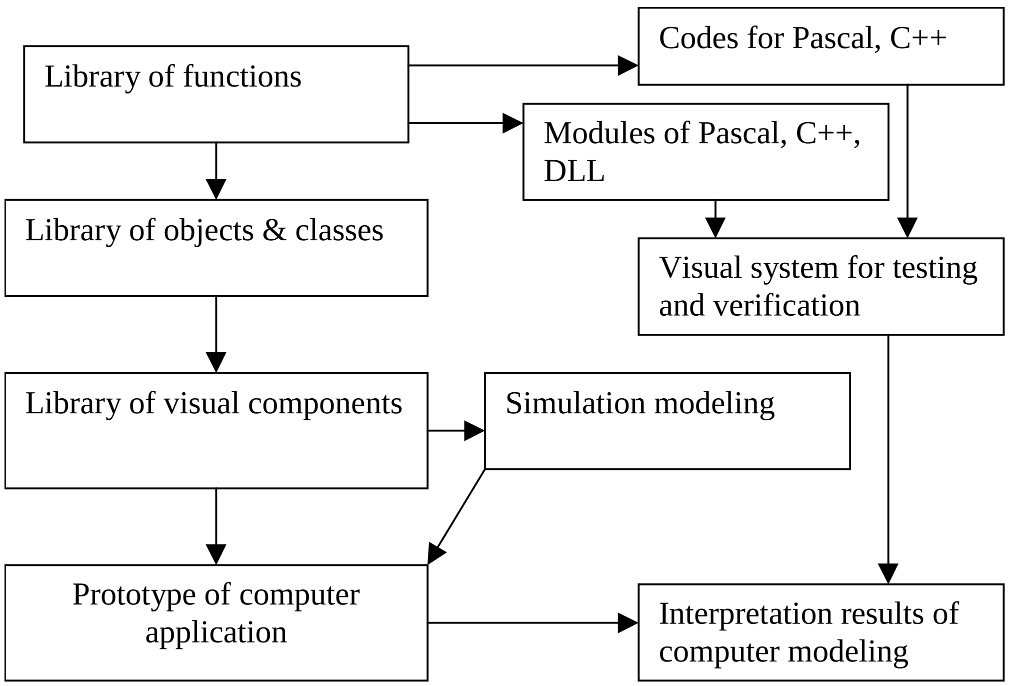


Fig.24. The proposed technology.

The CP is compiled as some domains.

The first part is a controlling object (supervisor). This object sends control messages to other objects and domains, and executes some other service functions.

The second part is a set of calculated functions and simulation models. They are used for modeling search problems and represent the main applied contents of TSMO. They can be used for solving some tasks as follows:

- Immediate modeling as a direct query to the library of functions. The library of functions is a basis for methods of applied objects and classes;
- Verification and testing of applied methods and utilities of objects and classes;
- Decision making for search operations at low level. It is recommend only for advanced users of CP that know theoretical bases of TSMO.

The third part is a database possessing information about Targets, Observers and Regions. As a rule search tasks are parts of some complex tasks. In this regard the information about Targets, Observers and Regions is very large. In real search problems only small portions of the whole information are used. According to this in the CP the database is built as a system of filters. The filters are selected and show only the information necessary for search operations. This reduces a complexity of the database and the user's informational loading.

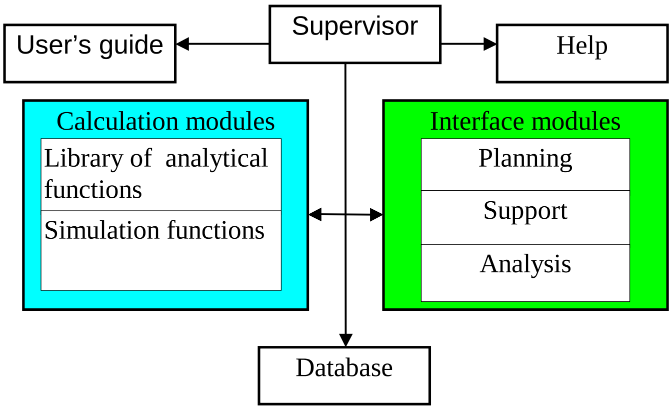


Fig.7. Module architecture of CP.

The forth part is a short manuals guide for the CP user, providing for some useful information about TSMO.

The fifth part is a conventional help system. This system presents users' information about search problems modeling on PC.

The sixth part is an interface modules for the CP users.

The main window of the CP is shown in Fig.26.

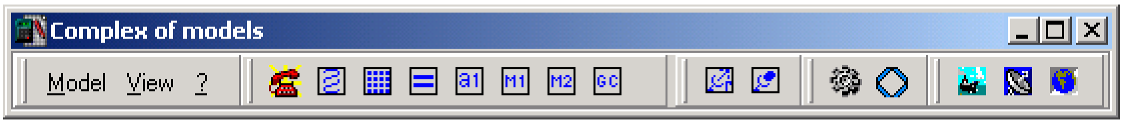


Fig.26. The main window of the CP.

3.2. Library of functions and visual components development.

The application technology of object-oriented modeling for computer modeling in various areas including complicated analytical models and theories is shown in fig.27. Visual library of functions is a basis of such an approach.

A structure of a given method is shown in fig.28. It is supposed, that decomposition of a subject area on the function base is performed, and the list of computable functions presented in a traditional for mathematical analysis form is available.

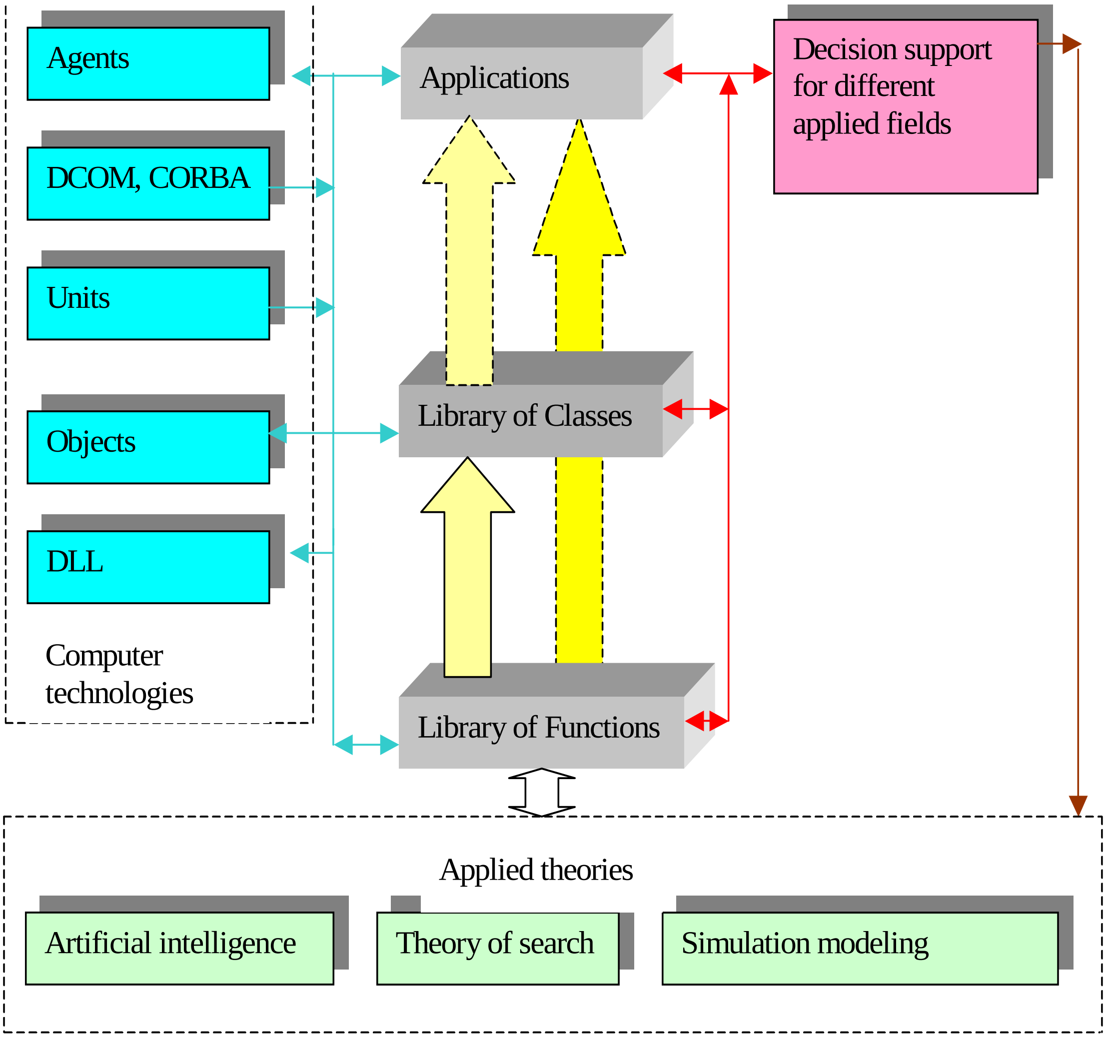


Fig. 27. Proposed technology for object-oriented modeling.

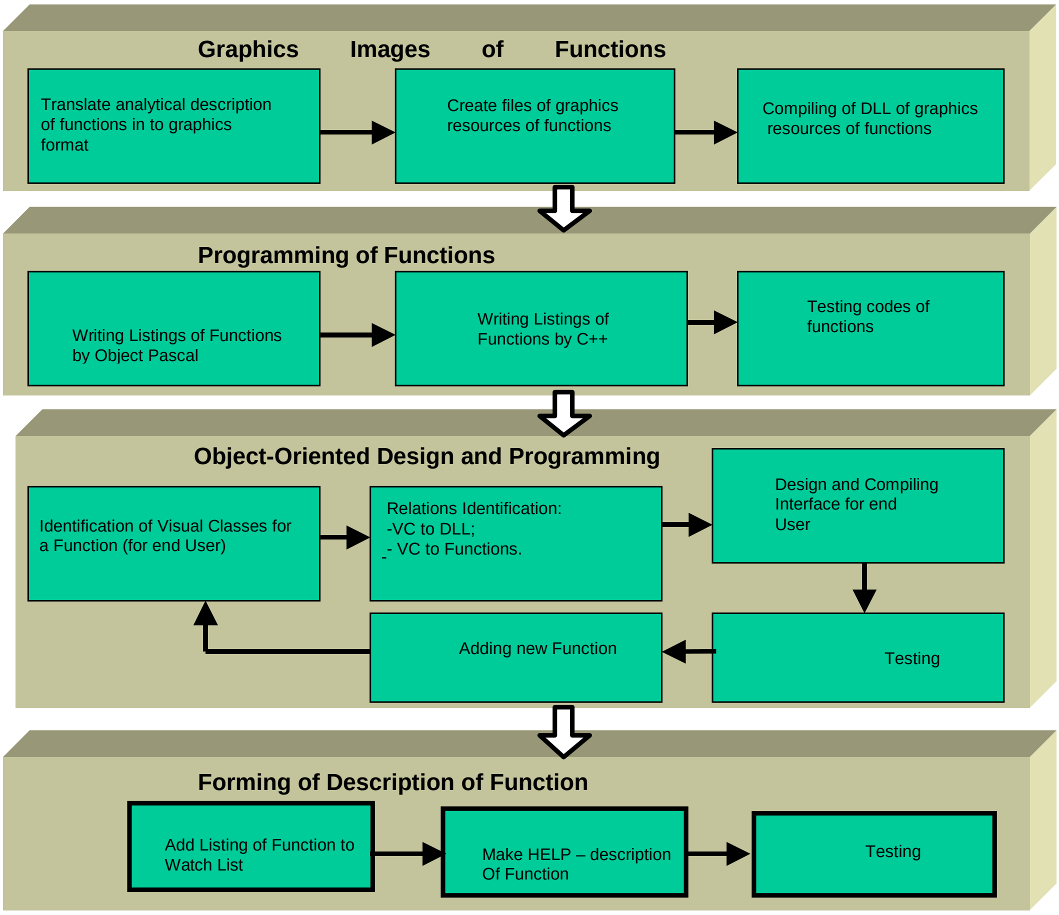


Fig.28 Structure of method of visual library of functions development.

An example of existing computer prototype is given in fig.29. The developed computer prototype allows to get a number of traditional functions intrinsic to computer libraries and a number of new functions; the principal functions among them are:

1. Visualization of a function in a traditional for mathematical analysis form in a main window (Fig. 29).

2. Function value calculation for a given vector of incoming arguments in a main window (Fig.29).

3. Function graph construction in a given range of arguments in a main window (Fig.30).

4. Function graph construction around any argument chosen by a user.

5. Production of function listings in Object Pascal and C++ languages (Fig. 31).

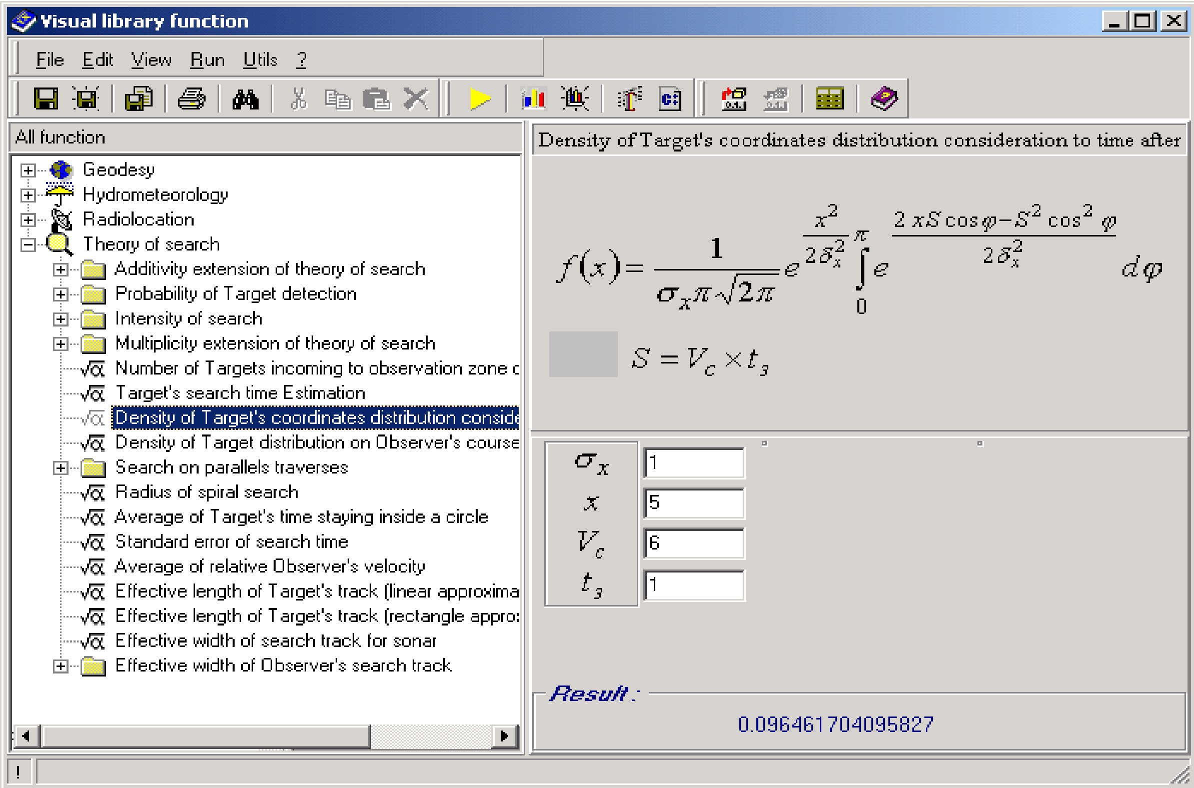


Fig. 29. Main window of visual library of functions.

Eleven functions of theory of search and six groups of elementary functions have been included in visual library as follows.

I. List of functions.
1). Number of Targets incoming to observation zone during the unit time.
2). Target’s search time estimation.
3). Density of Target’s coordinates distribution consideration to time after call.
4). Density of Target distribution on Observer’s courses.
5). Radius of spiral search.
6). Average of Target’s time staying inside a circle.
7). Standard error of search time.
8). Average of relative Observer’s velocity.
9). Effective length of Target’s track (linear approximation).
10). Effective length of Target’s track (rectangle approximation).
11). Effective width of search track for sonar.

II. List of groups of elementary functions.
1). Additivity extension of theory of search.
2). Probability of Target detection.
3). Intensity of search.
4). Multiplicity extension of theory of search.
5). Search on parallel beams.
6). Effective width of Observer’s search track.

About 10 search tasks have been developed as visual component:
1). Search by spiral.
2). Search after call.

3). Probability distribution density of the Targets by Observer azimuths.
4). Theorem of additivity OS1 and OS2.
5). Theorem of multiplicity OS1 and OS2 (the first definition).
6). Theorem of multiplicity OS1 and OS2 (the second definition).
7). Low-flying target detection range.
8). Simulation model for random search.
9). Simulation model for parallel search.
10). Data base for Target, Region and Observer.

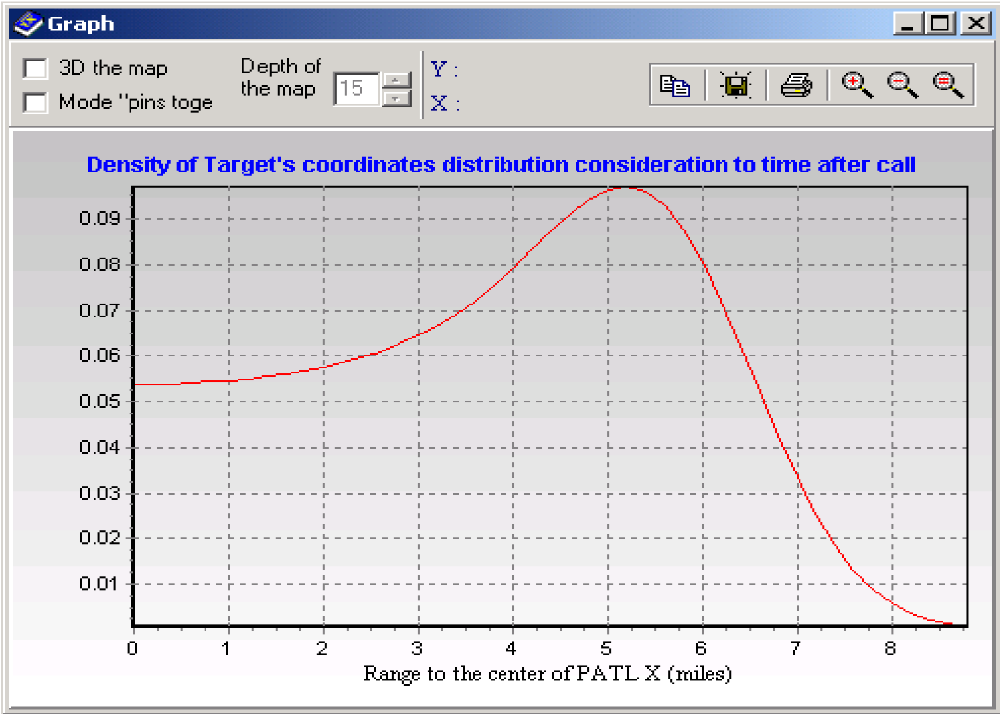
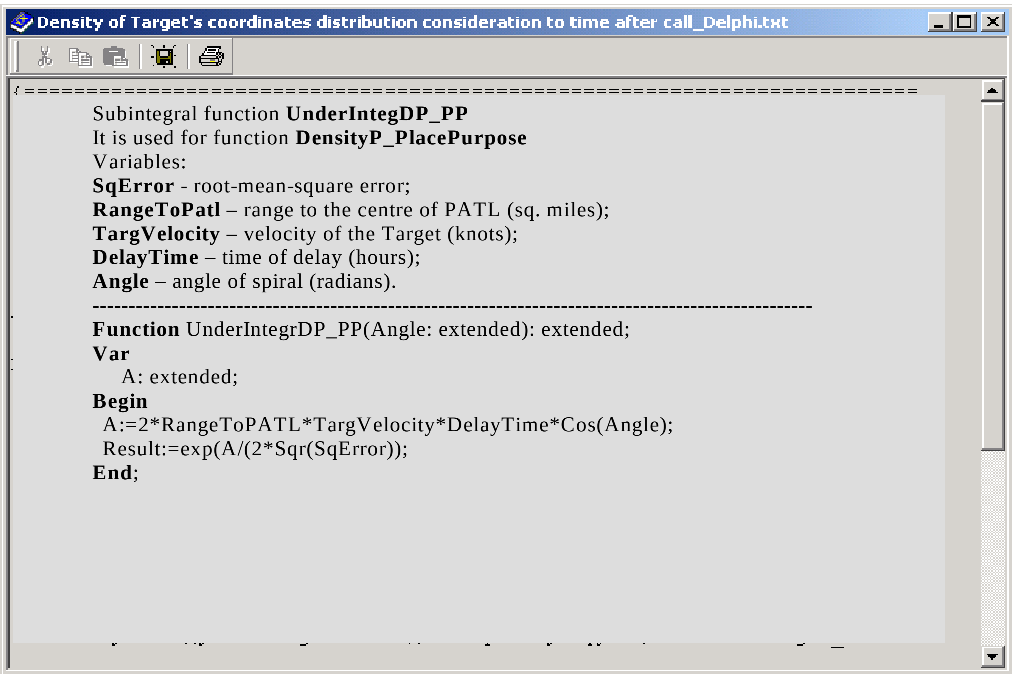


Fig. 30 Chart of selected function.




```
{ ===========================================================================
Subintegral function UnderIntegDP_PP
It is used for function DensityP_PlacePurpose
Variables:
SqError - root-mean-square error;
RangeToPatl – range to the centre of PATL (sq. miles);
TargVelocity – velocity of the Target (knots);
DelayTime – time of delay (hours);
Angle – angle of spiral (radians).
-----------------------------------------------------------------------------------------------
Function UnderIntegrDP_PP(Angle: extended): extended;
Var
   A: extended;
Begin
 A:=2*RangeToPATL*TargVelocity*DelayTime*Cos(Angle);
 Result:=exp(A/(2*Sqr(SqError));
End;
```

Fig. 31. Listing of selected function by Object Pascal.

3.3 *Prototyping and testing of computer models of search tasks.*

It is possible to show some examples how to use the proposed CP for the computer modeling. Some theorems of TS were selected as follows.

3.3.1. Probability distribution density of the Targets being detected by Observer azimuths.

This theorem has a good practical interpretation. One can understand the Observer's possibility to detect the Target based upon the results of computer modeling, depending on the correlation of Target's and Observer's velocities. The probability distribution density of the Targets being detected by Observer azimuths gives an information about detections in future. If an Observation System has limitation of azimuths (the observation zone is less than a circle), it will be possible to calculate a probability of the Target's appearance in the "shady zone", see Fig.32.

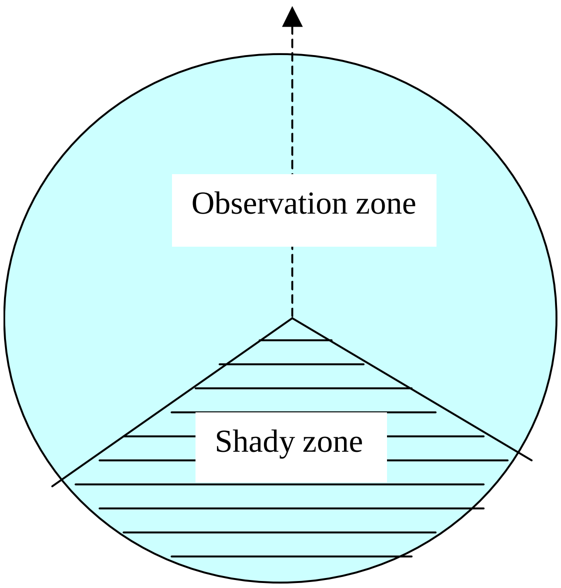


Fig.32. Observation and shady zones of the Observer.

The next formulas are used for the calculation and modeling:

$$\mathbf{1).\ V_n \le V_c\,.}$$

$$\mathbf{f(q) = \frac{1}{2\pi V_\rho}\left(V_n Cos^{-1}(-mCosq)Cosq + \sqrt{V_c^2 - V_n^2 Cos^2 q}\right)}$$

2). $\mathbf{V_n > V_c}$.

a). $\quad -\mathrm{Cos}^{-1}(m) \le q \le \mathrm{Cos}^{-1}(m);$

$$f(q) = \frac{V_n \mathrm{Cos}q}{2V_\rho}.$$

b). $\quad -\mathrm{Cos}^{-1}(-m) \le q \le -\mathrm{Cos}^{-1}(m);$

$\mathrm{Cos}^{-1}(-m) \le q \le \mathrm{Cos}^{-1}(-m);$

$$f(q) = \frac{1}{2\pi V_\rho}\left(V_n \mathrm{Cos}^{-1}(-m\mathrm{Cos}q)\mathrm{Cos}q + \sqrt{V_c^2 - V_n^2 \mathrm{Cos}^2 q}\right).$$

c). $\mathrm{Cos}^{-1}(-m) \le q \le -\mathrm{Cos}^{-1}(m);$

$f(q) = 0.$

where **$m = V_n/V_c$** .

The modal window of this theorem is shown in Fig.33.

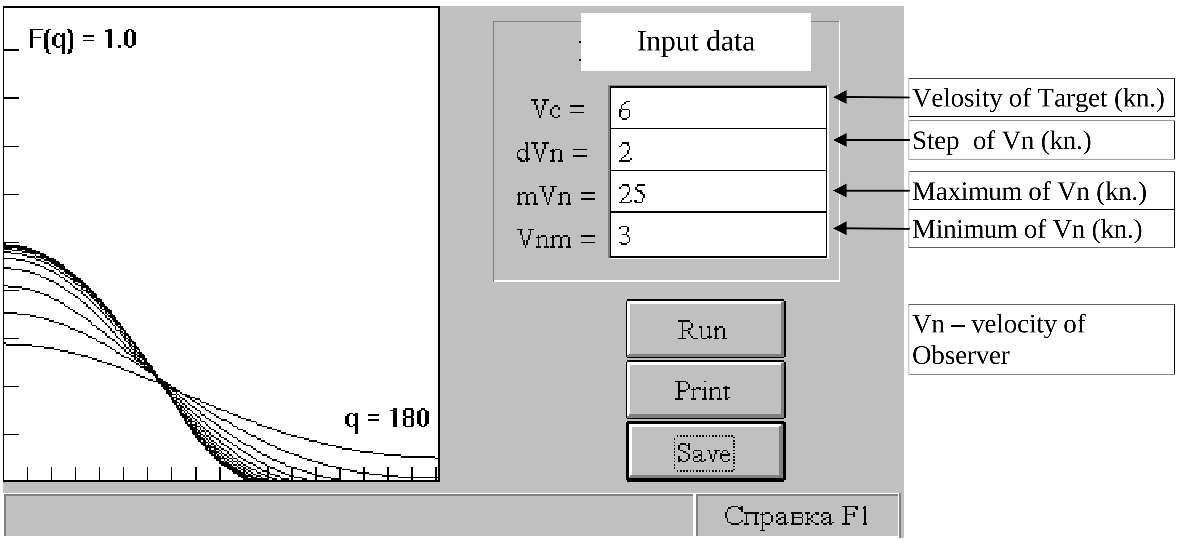


Fig. 33. The modal window of the task (probability distribution density of the Targets being detected by Observer azimuths).

3.3.2. Modeling of Observer track by spiral search.

The following formulas are used for calculation and modeling:

$$R_m = C \cdot e^{\frac{\varphi}{\sqrt{\frac{V_n^2}{V_c^2} - 1}}}, \qquad C = V_c \cdot T$$

The modal window of the task is shown in Fig.34.

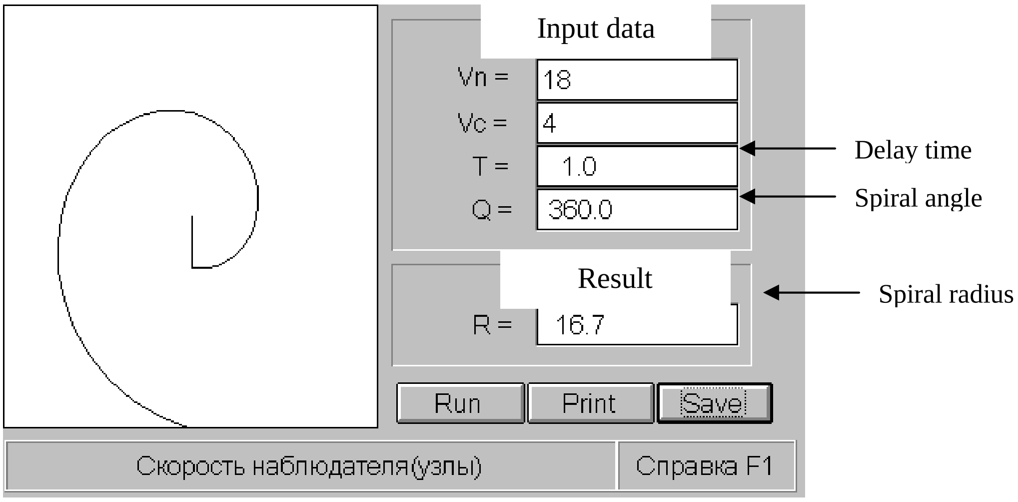


Fig.34. The modal window of the task (Modeling of Observer track by spiral search).

Similar computer interpretations for other theorems of theory of search for moving objects have been developed. Some search problems have been solved as examples.

3.3.3. Probability distribution density of target localization for delay time.

A main window of this task is shown below (Fig .35):

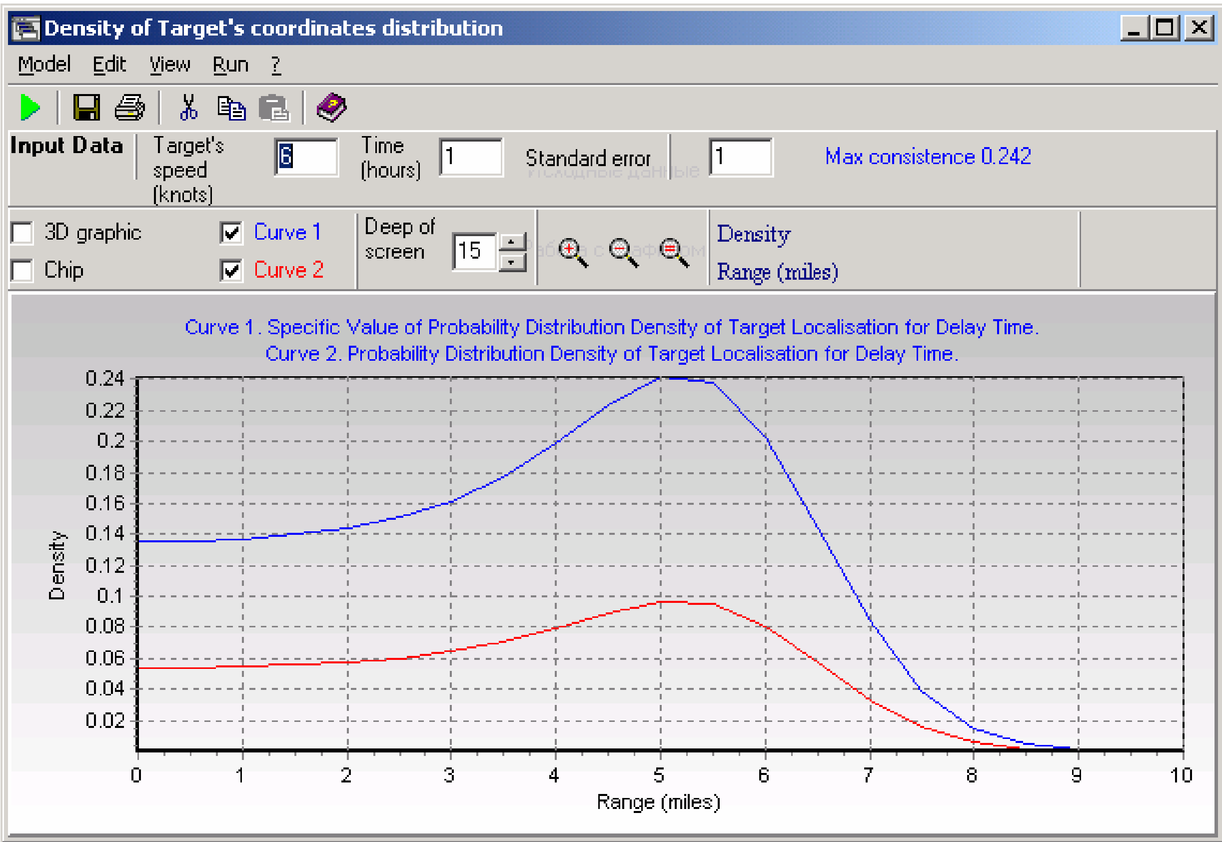


Рис. 35. User interface of «Probability distribution density of target localization for delay time» task.

3.3.4. Simulation model for random search.

Main window of this task is shown below (Fig .36). It is possible to solve some particular tasks that impossible to solve analytically.

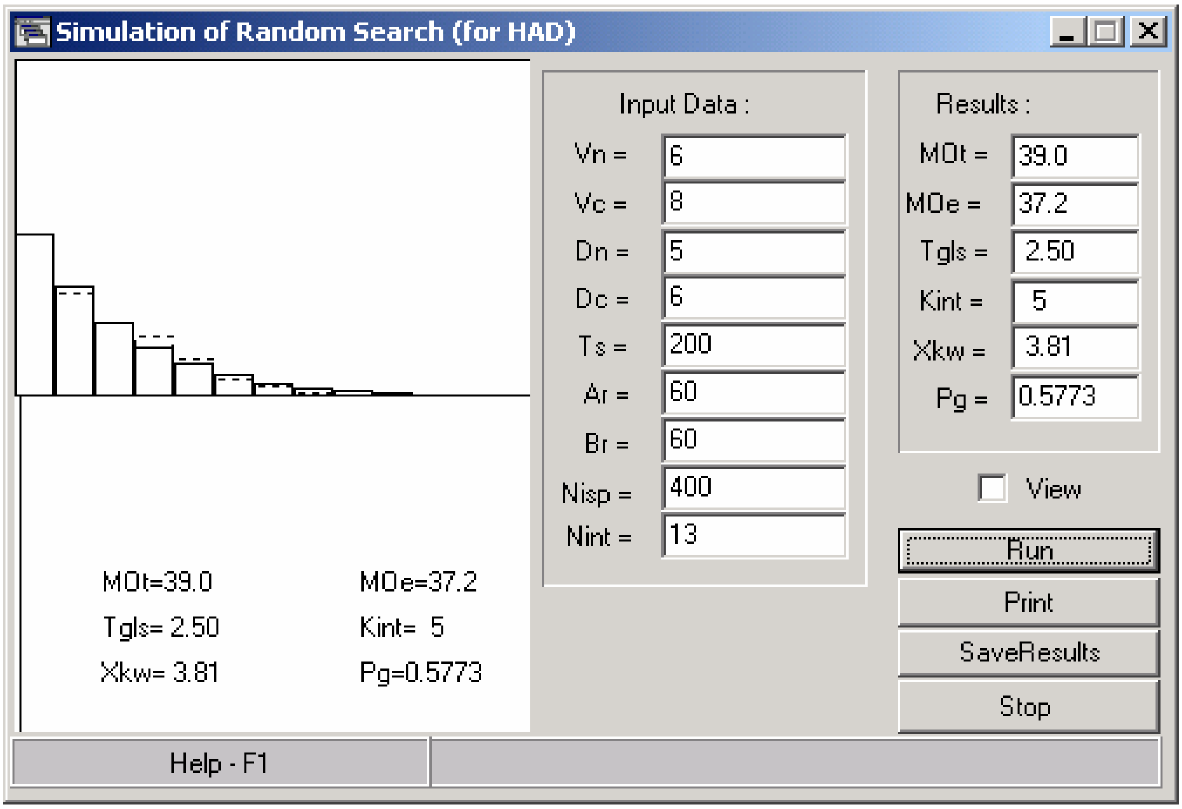


Fig .36. Simulation model for random search.

Testing of CP took three months. At this time 39 errors and bugs were found. Most of them were corrected. A test protocol is given in Appendix #1.

3.3.5. Solving some kind of search problems.

A problem of search optimization for aircraft incident on the Black Sea was considered. Common conditions at October 4, 2001 on the Black Sea are shown in Fig.1.

Initial situation:
October 4, 2001 year in 13.45 of Moscow time aircraft TU-154 of "Sibir" company was destroyed up Black sea. It was attacked by Ukrainian missile of system C-200 from military base "Opuk".

It was determined a probability region where the aircraft was shake down. Additional to that the start position of the missile and the position of missile sap were calculated. (Fig. 37)

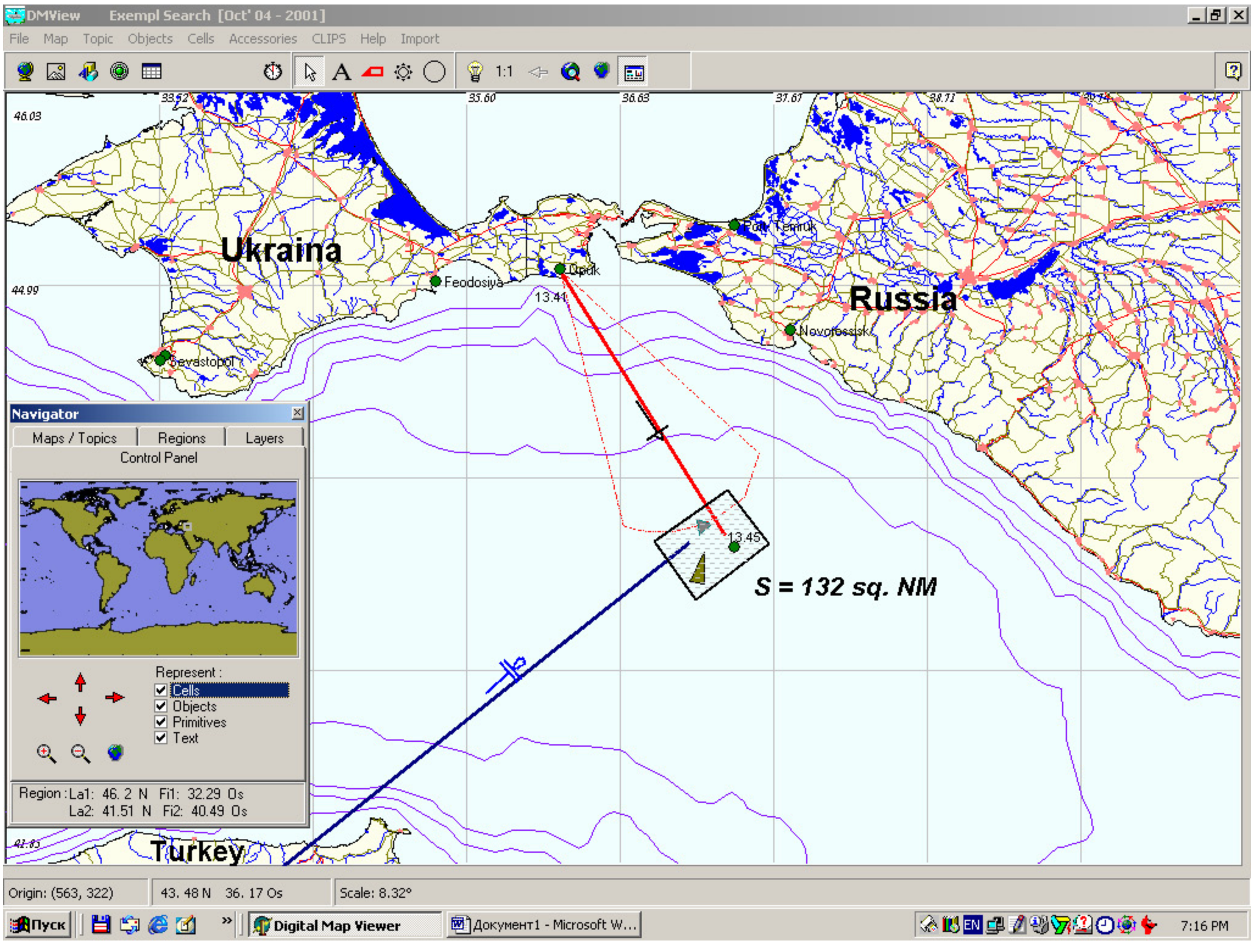


Fig.37. Common conditions on October 4, 2001 on the Black Sea.

Input data:

- dead-reckoning position of the aircraft at the moment of lost contact:
-

$$\varphi = 43^{\circ}14'2N \qquad \lambda = 37^{\circ}14'0E$$

- dead-reckoning position of the missile t:
-

$$\varphi = 43^{\circ}13'2N \qquad \lambda = 37^{\circ}17'0E$$

- observation point of hard parts of the aircraft:
-

$$\varphi = 43^{\circ}10'0N \qquad \lambda = 37^{\circ}16'5E$$

- observation point of the centre of oil spot:
-

$$\varphi = 43^{\circ}02'32N \qquad \lambda = 37^{\circ}18'0E$$

Region conditions.
Stream: course was 150 degrees, velocity was 6 knots.
Wind: course was 30 degrees, velocity was 15 meters per second.

According to the region conditions, input data, a search region was calculated as follows:

$$\varphi = 43^{\circ}08'2N \qquad \lambda = 37^{\circ}14'0E$$

$$\varphi = 43^{\circ}14'0N \qquad \lambda = 37^{\circ}10'1E$$

$$\varphi = 43^{\circ}02'3N \qquad \lambda = 37^{\circ}08'0E$$

$$\varphi = 43^{\circ}07'0N \qquad \lambda = 37^{\circ}04'4E$$

A square of the region was 132 sq. miles.

Input date for search operation are shown in Fig.38.

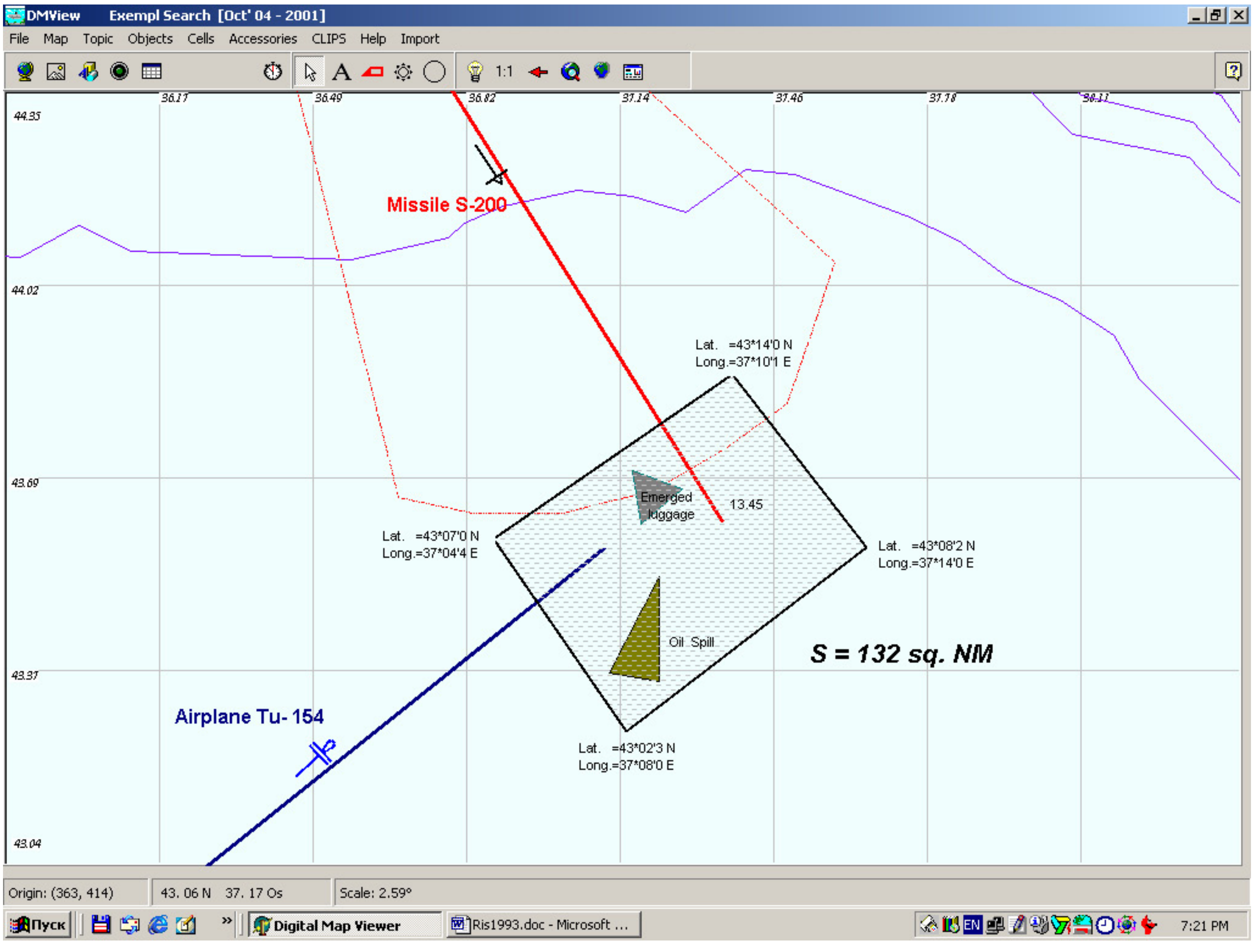


Fig..38. Input data for search operation.

Search effort for search operation:
- frigate of the Russian coast guard (Observer #1);
- ship "Svetogor" (Observer #2).

Observers are equipped with sonars; dependences of sonar range on velocity are shown in Fig.39 and Fig.40.

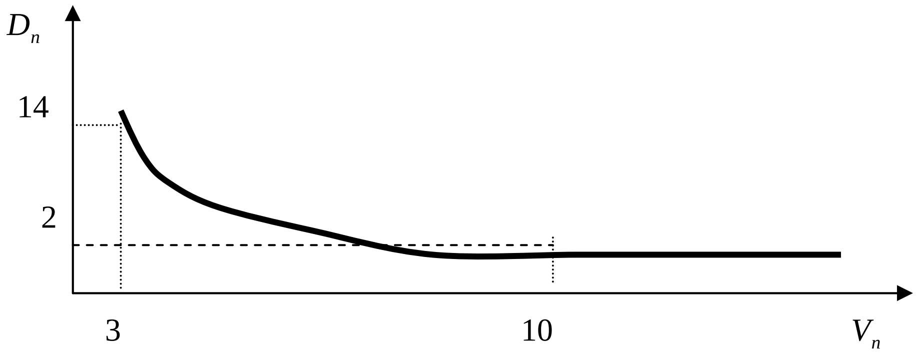


Fig.39. Dependence of sonar range on velocity for Observer #1.

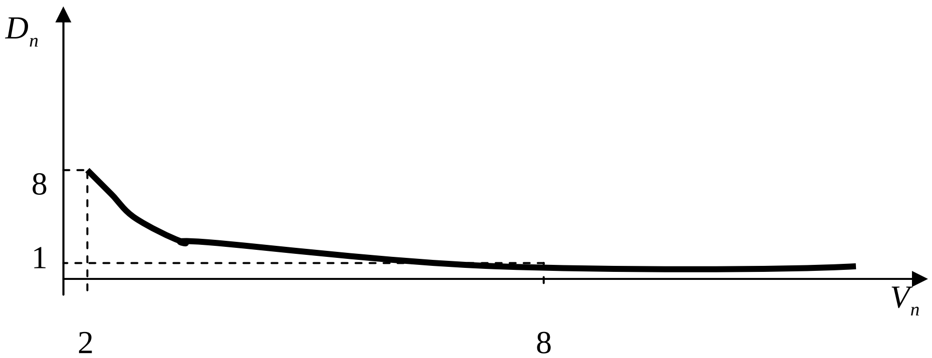


Fig. 40. Dependence of sonar range on velocity for Observer #2.

During the search a velocities were maximum, thus, the search time was not optimal.

For the maximum velocity:

T= S/ (D*Vn) = 132 / (2*((10*2)+(8*1)) = 2,35 hours = 2 hours 21 minutes.

Optimal velocities of Observers are: 6 and 5 knots accordingly. Search time for this is one hour six minutes.

Let us prove it.

For this practical problem we were solving a task: «Search of unmoving Target in the region by group of Observers».
Let us give a general description of the problem.
A group of Observers needs to perform a search of unmoving Target. Let us assume that coordinates of Target are uniformly distributed in a Region. Some parameters are known:
D_{ef} – effective width of search track of Observer;
V_n – velocity of Observer;
$D_{ef} = f(V_n)$;
S – square of search region.

It is needed to determine time of search and type of maneuvering.
A type of maneuvering is evidently – search by parallel tacks. This type allows to examine a region with any gaps. Random search in this case does not provide for 100% probability of

Target detection at any time of search, and average of time distribution will be smaller than for parallel tacks.

Average of distribution of search time can be determined as follows:

$$T = \frac{1}{2\gamma}; \qquad \gamma = \frac{D_{ef}V_n}{S};$$

Probability of target detection can be determined as follows :

$$P(t) = \frac{D_{ef}V_n}{S}t, \qquad (t \le T).$$

A full time of search can be determined as follows:

$$T = \frac{1}{\gamma}; \qquad P(t)_{t=T} = 1$$

For this task an optimization problem can be formulated as follows.
For real conditions an effective width of search is a function of velocity. It is not included in mathematical models of theory of search. But it is a fact that for some meanings of velocity a time of search will be minimal. Mathematically optimization problem can be formulated as follows:

$$T = \frac{S}{\sum_i (D_{ef}V_n)_i}; \qquad 0 < V_n \le V_{n\max}.$$

It is needed to determine: **T = min T{D(V_n), V_n)} .**

There is a typical task of calculus of variations for our problem – to obtain a maximum of functional.

Function **D(V_n)** for every Observer can be presented as shown in Fig.39 and Fig.40.

The function can be received as a result of testing under real conditions or as a result of simulation modeling. Then every function can be approximated by a continuous function. Let us look at a linear approximation for our case given in Fig.41.

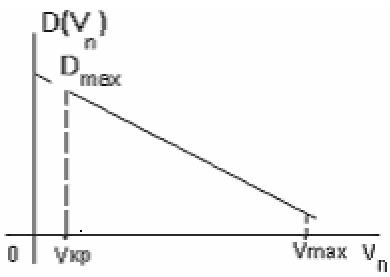


Fig. 41 Linear approximation of function **D(V_n)**

Equation of **D(V$_n$)** can be presented as follows:

$$D(V_n) = D_{\max} - (\frac{D_{\max} - D_{\min}}{V_{n_{\max}}} V_n)$$

Then for every I Observer we can write an equation as follows:

$$T = \frac{S}{V_n D_{\max} - V_n^2 \frac{D_{\max} - D_{\min}}{V_{n\max}}} = \frac{S \cdot V_{n\max}}{V_n D_{\max} V_{n\max} - V_n^2 \left(D_{\max} - D_{\min}\right)};$$

Functional **T** has **min** in a special point $\boldsymbol{\frac{dT}{dV_n}} = 0$ **.**

Let us solve an equation as follows:

$$\frac{dT}{dV_n} = \frac{d}{dV_n}\left(\frac{S \cdot V_{n\max}}{V_n D_{\max} V_{n\max} - V_n^2 \left(D_{\max} - D_{\min}\right)}\right);$$

After some transformations it is easy to receive the following:

$$V_n = \frac{D_{\max} \cdot V_{n\max}}{\left(D_{\max} - D_{\min}\right)}$$

An optimal meaning of Observer's velocity has been obtained. For this velocity time of search will be minimal.

For our input date and conditions (see above), optimal velocities are: $V_{n1} = 6$ knots and $V_{n2} = 5$ knots. For those velocities a time of search will be:

T = S/ (Def * Vn) = 132/ (2*((6*6,8)+(5*3,7)) = 1,11 hours = 1 hour 06 minutes.

Conclusion

The analysis of the advanced theory of search showed that the unified theory of search does not exist. In this regard it was proposed to develop the theory of search for moving objects. This theory does not study the complete set of well-known search tasks. and can be interpreted as the axiomatic theory. The proposed research was outlined as follows:
1.Axiomatic theory of search development.
2.Introduction of the new theorem (additivity and multiplicity).
3.Computational interpretation of theoretical results. Each theoretical result is developed to the level of a function or a method for PC.
4.Computer (object-oriented) interpretation of TSMO.

The main obtained research results were published in International conferences [31, 32]. A variant of Computer Prototype is attached to the report.

The Work Plan of scientific research has been completed. We have some proposals for future research as follows.

1. **DEVELOPMENT OF INFORMATION FUSION METHODS IN INTELLIGENT MULTIAGENT SYSTEM FOR SEARCH PROBLEMS.**

Results of research done for the project "New Class of Search Problems for Moving Objects" Project #1993p, Task #3 allow to arrive to a conclusion that the existence and interplay of four objects Observer, Target, Region and Operation form the core of the search problem, and the object-oriented approach is the best technique to solve the above problem. From the other point of view it is possible to consider Region as the most complex object, and Geo Information System (GIS) can be considered the best environment for a computer representation of a Region, and use of some novel approaches makes GIS a powerful interface for end users. It is expected that further search problems development on the basis of GIS will lead to significant advantages.
Capacity of GIS information is very large. And a problem how to save, to store, to use and fuse information is a very complex problem. Only technical part of the problem can be solved by OOP. According to this the Intelligent Agent Technology is the best way to receive a common solution for the problem of information fusion. Objectives of this project are:

- Multi-sensor data fusion for an observer (theorem of additivity and multiplicity) in GIS environment, level 1 of the JDL model;
- situation refinement for search operation under real conditions and real time (GIS), level 2 of the JDL model;
- impact assessment for search operation under real conditions and real time (GIS), level 3 of the JDL model;
- transform a system of classes that has been developed in project #1993p, Task #3 into a multi-agent intelligent system (MIS) for GIS.

Research results can possibly be applied to search of different kind of targets via air-born observation systems of various types operating under real conditions and in real time.

2. **DEVELOPMENT OF AXIOMATIC SEARCH THEORY FOR INTRUSION DETECTION IN COMPUTER NETWORKS.**

An axiomatic theory of search for moving objects was developed in project #1993. The search process in the theory is considered as a certain search operation including a number of objects such as: Observer, Target, Region and Search Operation proper. The bibliography of data and computer network protection against external (undesirable) intrusion shows that tasks are fairly similar in terms of objectives.

An address space of computer can be interpreted as one-dimensional, three-dimensional and multi-dimensional space. It depends on the items being examined: a hard disc, main memory, PC, local or global network. Such a representation is similar to a Region definition in the theory of search. In the given space laws of objects distribution can be selected or detected by experiment. The objects are targets of search. A search target is a run-time program, a file or an information (consequences of intrusion). An observer is a run-time program or a system incorporating hardware and a run-time program for a target detection. An operation is a system of activities in the Region arranged for a target detection, target consequences detection and Region protection against the intrusion of a target.

It is considered important not only to detect the fact of intrusion and its aftermath, but first of all to prevent such an intrusion. Currently standard-type hard- and software means of protection are designed to detect various types of intrusion disregarding many other factors. As a rule, such important factors as available resources (networks, time, computing resources) types of detection programs and strategy of their application are not taken into account. Similar problems have already been formulated and solved to a considerable degree in a different research field: theory of search of moving objects.

An objective of the present project is to develop a theoretical basis necessary for obtaining the qualitative and quantitative characteristics criteria and indices of effectiveness of search of intrusions in computer networks, and theoretical substantiation of requirements to detection programs and to computer networks protection procedures. The special significance of this project is seen in the sophisticated levels of program abstractions such as distributed objects (COM, CORBA), multi-agent systems, intelligent data processing and data fusion systems.

Results of research could be used for protection planning of computer networks of different levels and structure (Internet, Intranet, LAN, PC, COM, CORBA, multi-agent systems) against unauthorized access. Theoretical product could be used to look for different PC viruses and antiviral protection.

Abbreviations

A – axioms of the theory **.**
a – search region width (in miles).
b – search region length (in miles).
CP - computer prototype.
Dn – diameter of field for Observation system type #1.
Ds – length of Target track – field for Observation system type #2.
$\mathbf{D_{efc}}$ - effective width of a search track.
F – set of properties.
F(t) – the search potential of Observer.
G – a set of operation hypothesis.
H – theorems of the theory.
$\mathbf{K_c}$ - course of the target.
K – set of region auxiliary properties.
$\mathbf{K_n}$ – course of the observer.
L – a language of the theory.
OS – observation system.
OS1 – observation system type #1.
OS2 - observation system type #2.
OOA - object-oriented approach .
OOM - object-oriented model.
PATL – probability area of target localization.
S – a search region square (S=a*b).
S - a search region square (square miles).
TSMO - theory of search for moving objects.
TS - theory of search.
t – the search time (hours).
U – the search capacity of Observer (square miles per hour).
$\mathbf{V_n}$ – velocity of an observer.
$\mathbf{V_c}$ - velocity of the target.
γ - intensity of Target search.
W – effective projection of search track.

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Appendix 1. *Project # 1993P*

Task # 3

Test Protocol

Of the computer prototype (CP)

Interval of testing: 15 May 2002 - 20 August 2002.
Location: SPIIRAS.
Testers: Pavlov Y.F. , Ivakin Y.A.
Developers: Vasilev V.V.

N	Date	Model (task)	Discovered defects of the CP	Comments
1	2	3	4	5
1.	06.25.02	The library of functions (LF). Item “Service” of the main menu.	Commands «Save Result» and «Insert result» do not use Clip Board. It is impossible for end user to use obtained results in another applications.	Corrected 07.18.02 By Vasilev.
2.	07.02.02	LF. Item “Service” of the main menu. “Look”.	If «Tree - any Icons» set is Disabled it is impossible to change the set to Enable. End user must have a possibility to change the set both to Disable and Enable during the runtime period.	Corrected 07.15.02 By Vasilev.
3.	07.05.02	LF. Item “File” and the “Print” Speed Bar of the main menu.	It is possible to print only part of the image (when “Album” set is selected).	Corrected 07.17.02 By Ivakin and Vasilev.
4.	08.07.02	LF.	Values of datum lines without any comments..	Corrected 08.15.02 by Pavlov and Vasilev.
5.	07.09.02	LF.	Help string on the main form does not show a meaning of result’s value.	Corrected 07..02 by Vasilev.

N	Date	Model (task)	Discovered defects of the CP	Comments
1	2	3	4	5
6.	07.12.02	LF.	The set of LF's functions is less than theoretical set of functions of the theory of search for moving objects.	Corrected 08.14.02 by Ivakin and Vasilev.
7.	07.17.02	LF.	Two functions «Cut» and «Chip» are incompatible. When they are turn on together there are some distortions in the form.	Corrected 07.24.02 By Vasilev.
8	07.25.02	LF.	The functions for group search are absent in the LF.	Corrected 08.05.02 by Vasilev.
9.	08.01.02	LF. The function "Relative velocity of observer".	Listing in the show window does not correspond to the function.	Corrected 08.05.02 by Vasilev.
10.	08.08.02	LF. The function "Probability of target detection by parallel beams".	When γ >0,5 and a target's velocity is equal to zero the meaning of the probability of target detection is more than "1".	Corrected 08.14.02 By Vasilev.
11.	08.16.02	LF. The function "Observer search potential".	If: - Observer's width of the search track W_{ef} = 5; - Observer's relative velocity V_{ρ} = 20; - Region square S = 1000; - Time of search t = 4; There is a mistake in results of computer modeling.	Corrected 08.14.02 by Vasilev.
12.	08.16.02	LF. The function "Search after call".	Listing in the show window by C++ does not correspond to the function.	Corrected 08.20.02 By Ivakin and Vasilev.
13.	08.19.02	LF.	The font style and font size on the main and other windows must be changed.	Corrected 08.20.02 by Vasilev.

N	Date	Model (task)	Discovered defects of the CP	Comments
1	2	3	4	5
1.	07.19.02	System of the applied classes of the theory of search (SAC TS).	The system of the applied classes must be recompiled as a system of DCU and DLL modules.	Corrected 08.09.02 By Vasilev.
2.	07.19.02	SAC TS.	There are some mistakes in Source Code of SAC TS. All test cases were incorrect.	Corrected 07.31.02 by Ivakin and Vasilev.
3.	07.25.02	SAC TS.	Some applied functions in SAC TS are not the same as in the LF .	Corrected 08.07.02 by Ivakin and Vasilev.
4.	08.15.02	SAC TS.	The process of search operation synthesis is not included in the SAC TS.	Corrected 08.20.02 by Ivakin and Vasilev.
5.	08.15.02	SAC TS.	The main window of the application does not support standard accessories of the Windows systems such as: scrolling, moving, change size and so on.	Corrected 08.20.02 by Vasilev.

N	Date	Model (task)	Discovered defects of the CP	Comments
1	2	3	4	5
1.	05.16.02	Computer Prototype (CP). “Parallel traverses” model.	Datum lines “Probability” and “Time” do not correspond to the data on the show panel “Density” and “Range”.	Corrected 05.20.02 by Vasilev V.V.
2.	05.16.02	CP. “Spiral” model.	«Max Radius» value is not detected on the show panel.	Corrected 05.20.02 by Vasilev V.V.
3.	05.16.02	CP. «Hydro Acoustics Condition Analysis» model.	It is not possible to save and print results.	Corrected 05.20.02 by Vasilev V.V.
4.	05.17.02	CP. Simulation models.	Buttons **«View»** and **«Delay»** do not function correctly. There are some mistakes in an algorithm.	Corrected 07.18.02. By Pavlov Y.F. and Vasilev V.V.
5.	05.21.02	CP. “Location Range” model.	Buttons “Print” and “Save” do not work correct.	Corrected 05.30.02 by Vasilev V.V.
6.	05.21.02	CP. «Radar Visibility» model.	Buttons “Print” and “Save” do not work correctly.	Corrected 05.30.02 by Vasilev V.V.
7.	05.29.02	CP. «Radar Visibility» and “Location Range” models.	“Hint” of the entering date does not appear.	Corrected 08.08.02 By Ivakin Y.A. and Vasilev V.V.
8.	05.29.02	CP. «Density» model.	“F1” button does not function	Corrected 06.11.02 by Vasilev V.V.
9.	06.04.02	CP.	Listing comments are not clear.	Corrected 06.28.02 by Vasilev V.V.

N	Date	Model (task)	Discovered defects of the CP	Comments
1	2	3	4	5
10.	06.04.02	CP.	When window of object is selected and if field Clip Board is selected too, pushing the  button gives us same process as pushing the  button. As result there is “Form” or “Image” in the Clip Board. And it does not correspond to user’s setings.	Corrected 06.28.02 by Vasilev V.V.
11.	06.11.02	CP. «Radar Visibility» model.	Press  button in the table-graphics forms immediately turn on the standard Windows window. Printing is started automatically without any user’s settings.	Corrected 07.08.02 by Vasilev V.V.
12.	06.11.02	CP.	Help system does not correspond to computer models.	Corrected 08.08.02 by Ivakin Y.A. and Vasilev V.V.
13.	06.11.02	CP.	Multilanguage interface support does not work correctly.	Corrected 08.08.02 by Popovich V.V. and Vasilev V.V.
14.	06.19.02	CP. Simulation models.	Not possible to print the main form. Only entering data are being printed. Start of printing is turned on after pushing any buttons: “Cancel”,” Close”, “OK”.	Corrected 07.08.02 by Vasilev V.V.
15.	06.19.02	CP. Simulation models.	Help information is absent.	Corrected 06.25.02 By Ivakin and Vasilev

N	Date	Model (task)	Discovered defects of the CP	Comments
1.	2.	3.	4.	5.
16.	06.19.02	CP. Simulation models.	Results of simulation and mathematical modeling do not correspond to each other.	Corrected 08.15.02. By Pavlov and Vasilev.
17.	06.27.02	CP.	Help system does not work correctly.	Corrected 08.15.02 By Ivakin and Vasilev.
18.	07.23.02	CP. Simulation models.	Help system must be checked.	Corrected 08.02.02. By Pavlov, Ivakin and Vasilev.
19.	08.06.02	CP.	The models “Theorem of multiplicity (M1)” and “M2” have some mistakes.	Corrected 08.20.02 by Ivakin and Vasilev .
20.	08.16.02	CP. “Theorem of additivity” model.	The button “F1” of the main window does not function.	Corrected 08.20.02 Vasilev.

Testers:

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Ivakin Y.A.

“ “_______________2002.

Developer:

Vasilev V.V.

Task Principal Investigator:

Popovich V.V.

“ “_______________2002.